

American University of Beirut

STAT 230

Introduction to Probability and Random Variables

Fall 2009-2010

Final Exam

Exercise 1 Let X equal the weight of the soap in a 6-pound box. Assume the distribution of X is normally distributed with mean 6.05 and variance 0.0004

a) Find $P(X < 6.0171)$

b) If nine boxes of soap are selected at random from the production line, find the probability that at most two of the boxes weigh less than 6.0171 each

c) Find the probability that the mean weight of nine such boxes does not exceed 6.035

Exercise 2 Let (X, Y) be a couple of random variables with pdf

$$f(x, y) = 4/3 \quad , \quad 0 < x < 1 \quad , \quad x^3 < y < 1$$

a) Find the marginal pdf of X and Y . Are X and Y independent ?

b) Find $P(X > Y)$

Exercise 3 Let (X, Y) be a couple of random variables with joint pdf:

$$f(x, y) = x e^{-x(y+1)} \quad , \quad x > 0 \quad , \quad y > 0$$

Find the pdf of $Z = XY$.

Exercise 4 Let $X \rightsquigarrow \mathcal{N}(0, 1)$ be a standard normal random variable with pdf

$$f(x) = \frac{1}{\sqrt{2\pi}} \cdot e^{-x^2/2} \quad , \quad -\infty < x < \infty$$

Consider $Y = X^2$

a) Find the pdf of Y

b) Find $M_Y(t)$, the moment generating function of Y

Exercise 5 Three components are placed in series. The time in hours to failure of each has pdf

$$f(x) = \frac{x}{500^2} \cdot e^{-x/500} \quad , \quad 0 < x < \infty$$

Let $Y = \min(X_1, X_2, X_3)$.

a) Find the cdf of Y , the pdf of Y

b) Find $P(Y \leq 300)$

Exercise 6 In a collection of 40 batteries, 20 are of type A and 20 are of type B . Type A batteries last for an amount of time that has mean 50 and standard deviation 15; type B batteries last for an amount of time that has mean 30 and standard deviation 6. Approximate the probability that the total life of all 40 batteries exceeds 1700.