American University of Beirut

STAT 230

Introduction to Probability and Random Variables

Fall 2009-2010

quiz # 1 - solution

1. Twenty workers are to be assigned to 20 different jobs, one to each job. How many different assignments are possible ?

20!

2. Let A and B be two independent events with $P(A) = 1/3$ and $P(B) = 1/4$. Find $P(A|A\cup\overline{B})$

$P(A|A\cup\overline{B}) = \frac{P(A\cap(A\cup\overline{B}))}{P(A\cup\overline{B})} = \frac{P(A\cap\overline{B})}{P(A\cup\overline{B})} = \frac{P(A)P(\overline{B})}{P(A)+P(\overline{B})-P(A)P(\overline{B})} = \frac{(1/3).(3/4)}{1/3+3/4-1/4} = \frac{3}{10}$

3. Suppose we turn over cards simultaneously from two well shuffled decks of ordinary playing cards. We say we obtain an exact match on a particular turn if the same cards appears from each deck; for example, the queen of spades against the queen of spades. Find the probability of at least one exact match.

$p = 1 - \frac{1}{2!} + \frac{1}{3!} - \frac{1}{4!} + \ldots - \frac{1}{52!}$ (matching formula without replacement)

4. An excellent free shooter attempts several free throws until she misses; if $p = 0.9$ is the probability of making a free throw, find the probability that the third miss occurs on the 30th attempt.

$p = C_{29}^{2}(0.9)^{27}(0.1)^{3}$

5. Players A and B play a sequence of independent games. Player A throws a die first and wins on a "six". If he fails, B throws and wins on a "five" or "six". If he fails, A throws and wins on a "four", "five" or "six". And so on. Find the probability of each winning the sequence.

$P(A\ wins) = \frac{1}{6} + \frac{5}{6}.\frac{4}{6}.\frac{3}{6} + \frac{5}{6}.\frac{4}{6}.\frac{3}{6}.\frac{2}{6}.\frac{5}{6} = \frac{169}{324}$, and

$P(B\ wins) = 1 - \frac{169}{324} = \frac{155}{324}$

6. Find $P(\overline{B}|\overline{A})$, if $P(A|B) = 1$

$P(A|B) = 1$ means that B is included in A, then $\overline{B}$ is included in $\overline{A}$, and then $P(\overline{B}|\overline{A}) = 1$

7. An urn contains 8 white balls and 12 blue balls. One of the balls is drawn at random, if its white it is put back in the urn with 7 additional white balls, and is simply put back if its blue. A ball is then selected form the urn, find the probability that the first ball was blue given that the second ball drawn is also blue.

Let B_1 ={event the first ball is blue}, and B_2={event the second ball is blue},

$$P(B_1|B_2) = \frac{P(B_2|B_1) * P(B_1)}{P(B_2|B_1) * P(B_1) + P(B_2|\overline{B_1}) * P(\overline{B_1})} = \frac{\frac{12}{20}\cdot\frac{12}{20}}{\frac{12}{20}\cdot\frac{12}{20} + \frac{12}{27}\cdot\frac{8}{20}} = \frac{81}{121}$$

8. A purchaser of electrical components buys them in lots of size 10. It is his policy to inspect 3 components randomly from a lot and accept the lot only if all 3 are non defective. If 30 percent of the lots have 4 defective components and 70 percent have only 1, what proportion of lots does the purchaser reject?

$p(reject) = 1 - p(accept) = 1 - \left(0.3 * \frac{C_6^3}{C_{10}^3} + 0.7 * \frac{C_9^3}{C_{10}^3}\right) = 0.46$, hence the proportion is 46%

9. Five Americans, four French and three Russians participate in a chess tournament. In how many different ways can they be classified if only the nationality of the players matters ?

$$\frac{12!}{5!4!3!}$$

10. A, B, and C take turns in flipping a fair coin, A first, then B, then C, then A, and so on. The first one to get a head wins. Find the probability of winning for player A.

$P(A\ wins) = P(H) + P(TTTH) + P(TTTTTTH) + \ldots$

$$= \frac{1}{2} + \frac{1}{2^4} + \frac{1}{2^7} + \ldots = \sum_{n=0}^{\infty} \frac{1}{2}.\frac{1}{2^{3n}} = \frac{1}{2}\sum_{n=0}^{\infty} \frac{1}{8^n} = \frac{1}{2}.\frac{1}{1-\frac{1}{8}} = \frac{4}{7}$$

11. A person tosses a fair coin until a tail appears for the first time. If the tail appears on the nth flip, the person wins 2^n dollars. Let X denote the player's winnings. Show that $E(X) = +\infty$. This problem is known at the St. Petresburg paradox.

$X \rightsquigarrow \{2, 2^2, 2^3, \ldots, 2^n, \ldots\}$

$P(X = n) = \frac{1}{2^n}$

$E(X) = \sum_{n=1}^{\infty} 2^n.\frac{1}{2^n} = \sum_{n=1}^{\infty} 1 = +\infty$