Math 233 Final Exam

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1. Let X be a r.v. with the following p.d.f.

$$f(x) = \begin{cases} 1/3 & 0 < x < 1 \\ 1/3 & x = 2 \\ 1/3 & x = 3 \\ 0 & \text{elsewhere} \end{cases}$$

What is the d.f. of X, F(x)?

2. Let X and Y have the following joint p.d.f.:

(x, y)	(0, 0)	(0, 1)	(1, 0)	(1, 1)	(2, 0)	(2, 1)
f(x, y)	1/18	3/18	4/18	3/18	6/18	1/18

a). What is the marginal p.d.f. of X, i.e. $f_1(x)$?

b). What is the marginal p.d.f. of Y, i.e. $f_2(y)$?

c). What is $E(Y|X = 0)$?

d). What is $E(X|Y = 0)$?

3. Let X and Y have a bivariate normal distribution, $(X, Y) \sim N(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$. What is the distribution of $U = X + Y$?

4. Let $X_1, X_2, \ldots, X_n$ be mutually independent r.v., each with a b(1, p) distribution. Let $Y = \max(X_1, X_2, \ldots, X_n)$.

a). What are the possible values of Y?

b). What are the probabilities of the possible values of Y (i.e. the p.d.f. of Y)?

5. Let $X_1, X_2, \ldots, X_n$ be mutually independent r.v., each with a $\chi^2(1)$ distribution. Let $Z = X_1 + X_2 + \ldots + X_n$.

a). What is the distribution of Z ?

b). What are the $E[\{Z - n\}/\sqrt{(2n)}]$ and $\text{var}[\{Z - n\}/\sqrt{(2n)}]$?

6. A mouse is put in a maze or labyrinth in such a way that the mouse has three doors to choose from. Doors 1 and 2 lead the mouse back to where he started. Door 3 will lead him out of the maze and to food. Assume that the mouse chooses each door with equal probability and that the door the mouse chooses does not depend on any previous choice. The X be the r.v. that the mouse leaves the maze on his x^{th} choice, $x = 1, 2, 3, \ldots$.

a). What is the $Pr(X = x)$, for $x = 1, 2, 3, \ldots$?

b). What is the expected number of choices until the mouse leaves the maze?

c). What is the m.g.f. of X?

7. Let X be a r.v. having the possible values $\{3, 6, 9\}$, each with equal probability. Given $X = x$, a biased coin is tossed x independent times with probability of a head equal to p. Let Y be the r.v. equal to the number of heads on x tosses of the biased coin.

a). What is the p.d.f. of X?

b). What is the p.d.f. of $Y|X = x$?

c). What is the joint p.d.f. of X and Y?

d). What are E(Y) and var(Y)?

8. Let X and Y be independent r.v. each with an $N(\mu, \sigma^2)$ distribution.

a). What is the distribution of $V = X + Y$?

b). What is the distribution of $W = X - Y$?

c). Assume that V and W have a bivariate normal distribution. Are V and W independent?