



(2)

AMERICAN UNIVERSITY OF BEIRUT
MATH 233 FINAL EXAMINATION

Time = 2 hours

1. Six cards are drawn at random and without replacement from an ordinary deck of 52 cards.
 - (a) Find the joint density of the number of aces X and the number of kings Y .
 - (b) Find the conditional density of X given Y .
Hint: use the fact

$$\sum_{w=0}^k \binom{n_1}{w} \binom{n_2}{k-w} = \binom{n_1+n_2}{k}$$

2. Let X and Y be independent and identically distributed random variables with common density

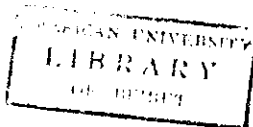
$$f(x) = \theta(1-\theta)^x \text{ for } x = 0, 1, 2, \dots$$

Find $P(X = Y)$.

3. The discrete density of the random variable X is given by $f(x) = x/3$ for $x = 1, 2$ and $f_{Y|X}(y|x)$ is binomial with parameters x and $1/2$; that is

$$f_{Y|X}(y|x) = \binom{x}{y} \left(\frac{1}{2}\right)^x \text{ for } y = 0, 1, \dots, x \text{ and } x = 1, 2$$

- (a) Find $E(X)$ and $Var(X)$.
- (b) Find $E(Y)$.
- (c) Find the joint distribution of X and Y .



4. Let X_1, X_2, \dots, X_n be n independent and identically distributed random variables with common density function

$$f(x) = x^{-2} \text{ for } x > 1.$$

Let $Y_1 = \min\{X_1, X_2, \dots, X_n\}$.

- (a) Does $E(X_i)$ exist? If so, Find it.
(b) Does $E(Y_1)$ exist? If so, Find it.
5. Let X_1 and X_2 be a random sample of size 2 from the density

$$f(x) = \frac{1}{2}e^{-\frac{1}{2}x} \text{ for } x > 0.$$

Find the distribution of the random variable $Y = X_1/X_2$.

6. Let X_1 and X_2 be a random sample of size 2 from the density

$$f(x) = 1 \text{ for } 0 < x < 1$$

Find the distribution of $Y = (X_1 + X_2)/2$.

7. Let X_1 and X_2 be a random sample from $N(0, 1)$; that is the common p.d.f. is $f(x) = (1/\sqrt{2\pi}) \exp(-1/2x^2)$ for all x

- (a) Show that $(X_1 + X_2)/\sqrt{2}$ and $(X_1 - X_2)/\sqrt{2}$ are independent and $N(0, 1)$ random variables.
(b) Find the distribution of $(X_1 + X_2)/(X_1 - X_2)$.