AMERICAN UNIVERSITY OF BEIRUT

MATH 233 FINAL EXAMINATION

Time = 1 hour 30 minutes

February 1, 1997

1. Let A and B be two events defined on the same sample space.
   (a) If $P(A) = .3$, $P(B) = .4$, and $P(A|B) = .5$, what is the conditional probability $P(A|B^*)$? (5pts)
   (b) Prove or disprove: If $P(A) = 3/4$ and $P(B) = 3/8$ then $1/8 \leq P(A \cap B) \leq 3/8$. (10pts)

2. A fair die is tossed 12 times.
   (a) Let X be the number of the face 6 and Y be the number of the face 1. Find the joint probability distribution of (X, Y). (10pts)
   (b) Find $COV(X, Y)$. (5pts)

3. The truncated Poisson distribution is defined as the following:
$$P(X = x) = \frac{\mu^x}{(e^\mu - 1)x!}; x = 1, 2, ...$$
   Find $E(X)$ and $Var(X)$. (10pts)

4. Given that the joint density of (X, Y) is
$$f(x, y) = \frac{k}{(1 + x + y)^3}; x > 0; y > 0$$
   (a) Find the constant k. (5pts)
   (b) Find the marginal density $f_1(x)$. (5pts)
   (c) Find the conditional density $f_{1|2}(x|y)$. (5pts)

5. Suppose that X and Y are independent random variables with the same $p.d.f.$ $f(x) = \theta e^{-\theta x}; x > 0$. Show that $X + Y$ and X/Y are independent and compute $E(X/Y)$. (15pts)

6. Let X and Y be independent random variable with densities $f(x) = (1 - x^2)^{-1/2}/\pi; -1 < x < 1$ and $g(y) = ye^{-y/2}/4; y > 0$ respectively. Find the distribution of the product random variable XY. (10pts)