

AMERICAN UNIVERSITY OF BEIRUT

MATH 233, Final Exam

February 2, 2002

Time = 1 hour 30 minutes

1. From an ordinary deck of 52 playing cards, cards are drawn one at a time and without replacement. Compute the probability that the third spade appears on the sixth draw. (5 pts)
2. Bowl I contains 7 red balls and 3 white balls and bowl II has 4 red and 6 white balls. Two balls are selected randomly and without replacement from Bowl I and transferred to bowl II. Three balls are then selected at random and without replacement from bowl II.
 - (a) Find the probability of selecting three white balls. (5 pts)
 - (b) Given three white balls are selected from bowl II, compute the conditional probability that two white balls were transferred from bowl I. (5 pts)
3. Let X and Y have the joint *pdf* $f(x, y) = kx^2y$ if $0 < x < y < 1$, zero elsewhere.
 - (a) What is the value of k such that $f(x, y)$ is a *pdf*? (5 pts)
 - (b) Find the value of the random variable $E(X|y)$. (10 pts)
4. Let X and Y be independent random variables with respective distributions $b(10, 1/3)$ and $b(15, 1/3)$. If $Z = 25 - X - Y$, what is the probability of Z exceeding 2? (10 pts)
5. It is known from the past that the birth weight (in Kg) of male baby in Lebanon has a $N(3.5, 1)$ distribution. Let Y denote the number of babies in a random sample of 20 new born male babies to exceed 3.5 kg.. Find $P(Y \leq 3)$. (10 pts)

6. Let X and Y be independent and identically distributed random variables with common distribution $N(0, 1)$. Let $U = X^2 + Y^2$ and $V = X + Y$.

(a) Find the joint *pdf*, $g(u, v)$, of (U, V) . (10 pts)

(b) Find the marginal density function of U . (5 pts)

(c) Compute the covariance of U and V . (5 pts)