AMERICAN UNIVERSITY of BEIRUT

STAT 230, Final Examination

Time = 1 hour and 50 minutes

Jan. 23, 2003

You are allowed to use one formula sheet, a table of discrete and continuous distributions, and a calculator!

1. Let A and B be two events such that $P(A|B) = P(A|B')$. Are the events A and B independent? (5 pts)

2. Let the *pdf*, $f(x)$, be positive at $x = -1, 0, 1$ and zero elsewhere.

   (a) If $f(0) = 1/4$, find $E(X^2)$. (5 pts)

   (b) If $f(0) = 1/4$ and if $E(X) = 1/4$, what is $M_X(t)$, the moment generating function, of X? (5 pts)

3. Can you find a random variable X which has a Poisson distribution such that $P(X = 0) = 2P(X = 1) = 8P(X = 2)$? (10 pts)

4. Consider an ordinary deck of 52 playing cards. Cards are drawn successively at random and one at a time. Find the probability of getting the fourth spade on the sixth draw if the sampling is done

   (a) with replacement. (5 pts)

   (b) without replacement. (5 pts)

5. Let X have a uniform distribution over the interval $(-1, 1)$. Define the transformation $Y = \ln(\frac{2}{X+1})^4$.

   (a) Find the *pdf* of Y. (10 pts)

   (b) Determine the mean and variance of Y. (5 pts)

6. Let the *pdf* $f(x) = 1/2$ if $0 < x < 1$ and $2 < x < 3$. Find the distribution function $F(x) = P(X \leq x)$. (10 pts)

7. Let $f(x) = \frac{k}{1+x^2}$ if $-\infty < x < \infty$.

   (a) For what value of the constant k $f(x)$ is a probability density function (*pdf*)? (5 pts)

   (b) Show that the mean value does not exist. (5 pts)s