AMERICAN UNIVERSITY OF BEIRUT
STATISTICS 230, Final Exam
Jan 31, 2005
Time = 1 Hour and 30 Minutes

You are allowed to use a formula sheet.

1. Let X be a discrete random variable that assumes positive probabilities on the set $\{1, 2, \cdots\}$. Show that $E(X) = \sum_{x=0}^{\infty}[1 - F_X(x)]$, when it exists.

2. Let X and Y be two random variables with a joint cumulative distribution function $F(x, y) = P(X \leq x, Y \leq y)$. Show that $F_X(x) + F_Y(y) - 1 \leq F(x, y) \leq \sqrt{F_X(x)F_y(y)}$, where $F_X(x)$ and $F_Y(y)$ are the respective cumulative distribution functions of the random variables X and Y.

3. Let $X_1, X_2, \cdots, X_n$ be n independent and identically distributed random variables with common probability density function $f(x) = ke^{-|x-\theta|}$, if $-\infty < x < \infty$.

   (a) Determine the value of k such that $f(x)$ is a probability density function.

   (b) Find $\hat{\theta}$, the maximum likelihood estimate of θ. Hint: You may need to minimize $\sum_{1}^{n} |X_i - \theta|$

   (c) if $n = 3$, is the estimator found in (2) unbiased?

4. Let X denote the time required to do a computation using algorithm written in programming language A, and let Y denote the time required to the same calculations using programming language B. Assume further that X is normally distributed and with mean 10 seconds and standard deviation 3 seconds and Y is normally distributed with mean 9 seconds and standard deviation of 4 seconds.

   (a) What is the distribution of $X - Y$?

   (b) Find the probability that a given calculation will run faster using A than when using B.

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