STAT 230
Final Exam
Feb 1, 2006
Time = 1 hour

1. Let $\underline{X} = (x_1, \cdots, x_m)'$ be a random sample from the *pdf*

$$f(x;\sigma) = \frac{1}{\sigma\sqrt{2\pi}} \exp[-\frac{1}{2\sigma^2}(x-10)^2]$$

If $\sum_{i=1}^{20} X_i = 180$ and $\sum_{i=1}^{20} X_i^2 = 2000$, find a 90% confidence interval for σ.

2. Let X be a Bernoulli random variable with a *pdf*

$$f(x;\theta) = \theta^x(1-\theta)^{1-x} \text{ if } x = 0 \text{ or } 1 \text{ and } 0 < \theta < 1$$

We would like to test the hypothesis

$$H_0 : \theta = \frac{1}{2} \text{ vs } H_a : \theta = \frac{2}{3}$$

It is agreed to perform two observations X_1 and X_2 . If both $X_1 = 1$ and $X_2 = 1$ then we reject H_0; otherwise we don't reject H_0. Find $\alpha = P(\text{Type I error})$ and $\beta = P(\text{Type II error})$.

3. If X has the *pdf*

$$f(x;\mu) = \frac{1}{10\sqrt{2\pi}} \exp[-\frac{1}{2}(\frac{x-\mu}{10})^2],$$

How large should n be chosen so that when testing $H_0 : \mu = 100$ vs $H_a : \mu = 110$ the values of $\alpha = 0.05$ and $\beta = 0.10$?

4. A box is known to contain either 3 red and 5 black balls or 5 red and 3 black balls. Three balls are drawn randomly and without replacement. If three red balls are obtained, the decision will be 5 red and 3 black; otherwise, the decision will be 3 red and 5 black balls. Calculate the values of α and β.