AMERICAN UNIVERSITY of BEIRUT

STAT 230, Final Examination
Time = 1 hour and 30 minutes
Jan 25, 2008

You are allowed to use a calculator, one formula sheet, tables of discrete and continuous distributions. (GOOD LUCK!)

1. Consider the experiment of rolling a pair of fair 4-sided dice. Let X_1 be the outcome on die 1 , and X_2 be the outcome on die 2 respectively. Define $Y_1 = \max\{X_1, X_2\}$ and $Y_2 = |X_1 - X_2|$. Determine the Covariance of Y_1 and Y_2. [15 pts]

2. Let the joint *pdf* of the two random variables X and Y be $f(x, y) = kx$ if $0 < x < y < 1$, 0 elsewhere, and k is a positive constant.
   (a) Find the value of k such that $f(x, y)$ is a joint *pdf*. [5 pts]
   (b) Determine the marginal *pdf* of X, $f_1(x)$, and the marginal *pdf* of Y, $f_2(y)$, respectively. [10 pts]
   (c) Are the random variables X and Y independent? [5 pts]

3. Let X_1 and X_2 be a random sample of size 2 from the standard normal distribution. Let $Y_1 = (X_1 + X_2)/\sqrt{2}$ and $Y_2 = (X_1 - X_2)/\sqrt{2}$. Show that Y_1 and Y_2 are independent standard normal random variables. Hint: Try to find the joint mgf of $(Y_1, Y_2) = E(e^{\{t_1Y_1+t_2Y_2\}})$. [15 pts]

4. Let X_1 and X_2 be a random sample from the exponential distribution with common *pdf* $f(x) = e^{-x}$ if $x > 0$. Let $Y_1 = X_1/X_2$ and $Y_2 = X_1 + X_2$.
   (a) Find the joint distribution of (Y_1, Y_2). [10 pts]
   (b) Find the mean and variance of Y_1. [10 pts]

5. The "Fill" problem is important in many industries, like those making cereal, tooth paste and so on. If such an industry claims that it is selling 100 grams of its product in a container or else the FDA will crack down. Although the FDA will allow a very few percentage to have a fill of less than 100 grams.

   (a) If the contents X of a container has a normal distribution with mean $\mu = 100.01$ and an unknown variance of σ^2. Find the value of σ^2 if $P(X < 100) = 0.025$ [10 pts]

   (b) In 50 independent such measurements, approximate the probability that at least one container is under 100 grams of fill. [5 pts]

6. In the casino game of Craps, the probability of winning a bet is 0.493. Let Y denote the number of winnings in a series of 1000 bets that are placed. Approximate $P(Y > 500)$. [15 pts]