American University of Beirut STAT 230 Introduction to Probability and Random Variables

Fall 2007-2008

Final Exam

Name:

ID #:

Exercise 1 Let X_1, X_2, X_3 be three independent random variables with binomial distributions b(4, 1/2), b(6, 1/3), and b(12, 1/6) respectively.

- **a.** find $P(X_1 = 2, X_2 = 2, X_3 = 5)$
- **b.** find $E(X_1X_2X_3)$
- **c.** find the mean and the variance of $Y = X_1 + X_2$

Exercise 2 The joint pdf of two random variables X and Y is

$$f(x, y) = kx \qquad 0 < x < y < 1$$

- **a.** find the value of the constant k
- **b.** find the marginal pdf of X and Y. Are they independent?
- c. find P(X + Y < 1/2)
- **d.** find $E(X^2Y)$

Exercise 3 A man and a woman decide to meet at a certain location. If each person independently arrives at a time uniformly distributed between 12 noon and 1 PM, find the probability that the first to arrive has to wait no longer than 10 minutes.

Exercise 4 Let X and Y be a random sample of from the exponential distribution with pdf

$$f(x) = e^{-x} \qquad 0 < x < \infty$$

Let U = X/Y and V = X + Y. Find the joint pdf of the couple (U, V). Are U and V independent?

Exercise 5 Let X_1, X_2, \ldots, X_n be a random sample of size n with uniform distribution over (0, 1). Find the pdf of $Y = \max(X_1, X_2, \ldots, X_n)$. Find E(Y), Var(Y) and $M_Y(t)$, the moment generating function of Y.