



American University of Beirut  
Math 233  
Final Exam  
June 16, 1997  
Time=1 hr 30 minutes

1. Let  $f(x) = (1/\beta)[1 - |(x - \alpha)/\beta|]$  for  $\alpha - \beta < x < \alpha + \beta$  where  $\alpha$  and  $\beta$  are fixed constants satisfying  $-\infty < \alpha < \infty$  and  $\beta > 0$ .
  - (a) Demonstrate that  $f(x)$  is a *pdf* of  $X$ .(5pts)
  - (b) Find the cumulative distribution function (cdf) of  $X$ .(5pts)
  - (c) Find the mean and the variance of  $X$ . (10pts)
2. A fair coin is tossed four times. Let  $X$  be the number of times a head is followed by tail. Find the distribution, mean and variance of  $X$ .(10pts)
3. Let  $X$  and  $Y$  be two random variables for which  $V(X) = 24$ ,  $V(Y) = 54$ , and the correlation coefficient between  $X$  and  $Y$ ,  $\rho = -0.5$ . What is the variance of  $X - Y$ ? (10pts)
4. Define random variables  $S = X + Y$  and  $D = X - Y$ , where  $X$  and  $Y$  are the values obtained on the first and second rolls of a balanced die. Find the mean and variance of  $S$  and  $D$  and show that  $S$  and  $D$  are uncorrelated. Are  $S$  and  $D$  independent? Explain briefly. (10pts)
5. Let  $X$  and  $Y$  be independent variables, each having the same geometric distribution i.e.  $f(x) = p(1 - p)^x$  for  $x = 0, 1, 2, \dots$ . Find  $P(X = Y)$ .(10pts)
6. Suppose  $X$  and  $Y$  are independent random variables, each is uniformly distributed over the interval  $(0,1)$ . Find the distribution of the two random variables  $U = XY$  and  $V = X/Y$  respectively.(20pts)

