Math 233 Spring 1998 Final Exam

1. Let X be a r.v. with the following p.d.f. $f(x) = \begin{cases} 1/4 & -1 < x < 0 \\ 1/2 & x = 0 \\ 1/4 & 0 < x < 1 \\ 0 & \text{elsewhere} \end{cases}$

What is the d.f. of X, F(x) ?

2. Let X be a r.v. with the following d.f. $F(x) = \begin{cases} 0 & 0 < x \\ x^2/2 & 0 \le x < 1 \\ 1/2 & 1 \le x < 2 \\ 1 & 2 \le x \end{cases}$

What is the p.d.f. of X, f(x) ?

3. Let X and Y have the following joint p.d.f.; $f(x, y) = \begin{cases} \dfrac{x y e^{-x}}{3} & 0 < y < x,\ 0 < x < \infty \\ 0 & \text{elsewhere} \end{cases}$

a). What is the marginal p.d.f. of X, i.e. $f_1(x)$?
b). What is the marginal p.d.f. of Y, i.e. $f_2(y)$?
c). What is the $E(Y|X)$?
d). What is the E(Y) ?

4. Let X and Y be independent r.v. each with an $N(\mu, \sigma^2)$ distribution.

a). What is the distribution of $V = X + Y$?
b). What is the distribution of $W = X - Y$?
c). Assume that V and W have a bivariate normal distribution. Are V and W independent?

5. Let $X_1, X_2, \ldots, X_n$ be mutually independent r.v.'s, each with a $\Gamma(1, 1)$ distribution.

a). Identify the distribution of $Z = X_1 + X_2 + \ldots + X_n$.

Let $Y = \max(X_1, X_2, \ldots, X_n)$.

b). What is the d.f. of Y?
c). What is the p.d.f. of Y?

6. A cat chases a mouse through the streets of a city. The cat can only catch the mouse, with probability 1/2, when the mouse stops to cross a street. The chase continues until the cat catches the mouse.

a). What is the probability that the cat catches the mouse on the n^{th} street crossing, $n = 1, 2, 3, \ldots$?
b). What is the probability that the cat catches the mouse?
c). What is the expected number of street crossings in the chase?

7. Let $X \sim N(\mu, \sigma^2)$. What is the distribution of $Y = (X - \mu)^2$? Show all work.

8. Let $X \sim b(n, p)$, where n is a realization or experimental value of the r.v. N which has a Poisson distribution with parameter λ.

a). What is the $E(X|N=n)$?
b). What is the $var(X|N=n)$?
c). What is the $E(X)$?
d). What is the $var(X)$?