

1. Let X be a r.v. with the following p.d.f.
- $$f(x) = \begin{cases} 1/4 & -1 < x < 0 \\ 1/2 & x = 0 \\ 1/4 & 0 < x < 1 \\ 0 & \text{elsewhere} \end{cases}$$

What is the d.f. of X , $F(x)$?

2. Let X be a r.v. with the following d.f.
- $$F(x) = \begin{cases} 0 & 0 < x \\ x^2/2 & 0 \leq x < 1 \\ 1/2 & 1 \leq x < 2 \\ 1 & 2 \leq x \end{cases}$$

What is the p.d.f. of X , $f(x)$?

3. Let X and Y have the following joint p.d.f.;
- $$f(x, y) = \begin{cases} \frac{xye^{-x}}{3} & 0 < y < x, 0 < x < \infty \\ 0 & \text{elsewhere} \end{cases}$$

- What is the marginal p.d.f. of X , i.e. $f_1(x)$?
- What is the marginal p.d.f. of Y , i.e. $f_2(y)$?
- What is the $E(Y|X)$?
- What is the $E(Y)$?

4. Let X and Y be independent r.v. each with an $N(\mu, \sigma^2)$ distribution.

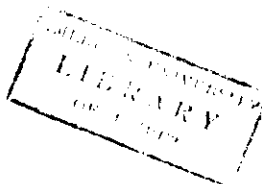
- What is the distribution of $V = X + Y$?
- What is the distribution of $W = X - Y$?
- Assume that V and W have a bivariate normal distribution. Are V and W independent?

5. Let X_1, X_2, \dots, X_n be mutually independent r.v.'s, each with a $\Gamma(1, 1)$ distribution.

- Identify the distribution of $Z = X_1 + X_2 + \dots + X_n$.

Let $Y = \max(X_1, X_2, \dots, X_n)$.

- What is the d.f. of Y ?
- What is the p.d.f. of Y ?



6. A cat chases a mouse through the streets of a city. The cat can only catch the mouse, with probability $1/2$, when the mouse stops to cross a street. The chase continues until the cat catches the mouse.

- a). What is the probability that the cat catches the mouse on the n^{th} street crossing, $n = 1, 2, 3, \dots$?
- b). What is the probability that the cat catches the mouse?
- c). What is the expected number of street crossings in the chase?

7. Let $X \sim N(\mu, \sigma^2)$. What is the distribution of $Y = (X - \mu)^2$? Show all work.

8. Let $X \sim b(n, p)$, where n is a realization or experimental value of the r.v. N which has a Poisson distribution with parameter λ .

- a). What is the $E(X|N=n)$?
- b). What is the $\text{var}(X|N=n)$?
- c). What is the $E(X)$?
- d). What is the $\text{var}(X)$?