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Math 233 Final Exam

1. (13 pts.) Consider a sequence of n nonoverlapping finitely small intervals. Changes that occur in the n intervals are independent and have the same probability of occurrence given by, where X = # occurrences in an interval, $0 < p, q, r < 1$ and $p + q + r = 1$;

$$P(X = 1) = p$$
$$P(X = 2) = q$$
$$P(X = 3) = r.$$

Find the following probabilities.

a). $P(\text{total occurrences in the } n \text{ intervals} = n)$.
b). $P(\text{total occurrences in the } n \text{ intervals} = n + 1)$.
c). $P(\text{total occurrences in the } n \text{ intervals} = n + 2)$.
d). $P(\text{total occurrences in the } n \text{ intervals} > n)$.

2. (10 pts.) Consider the following p.d.f. of a r.v. X.

$$f(x) = \begin{cases} x/2 & 0 < x < 1 \\ 1/2 & x = 1 \\ x/6 & 1 < x < 2 \\ 0 & \text{elsewhere} \end{cases}$$

What is the d.f., $F(x) = P(X \leq x)$, of X?

3. (10 pts.) Let the r.v. X have the following d.f.

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{x+1}{4} & 0 \leq x < 2 \\ 1 - \frac{e^{2-x}}{4} & x \geq 2. \end{cases}$$

What is the p.d.f. of X, $f(x)$?

4. (7 pts.) Let X be a r.v. such that $E(X^r) = r!$, $r = 0, 1, 2, \ldots$

a). Find the m.g.f. of X.
b). What is the distribution of X?

5. (8 pts.) Let Y be a r.v. such that $E(Y^r) = \frac{1}{r+1}$, $r = 0, 1, 2, \ldots$

a). Find the m.g.f. of Y.
b). What is the distribution of Y?

6. (20 pts.) Let X be a r.v. with the following p.d.f.

$$f(x) = \begin{cases} \frac{3}{16}x^2 & -2 < x < 2 \\ 0 & \text{elsewhere.} \end{cases}$$

a). What is the p.d.f. of the r.v. $Y = X^2$?
b). What is the p.d.f. of the r.v. $V = |X|$?

7. (20 pts.) Let X have a Poisson distribution with parameter λn. Provide the details of the proof to show that the Central Limit Theorem is true, that is, show that $W = \frac{X-\lambda n}{\sqrt{\lambda n}}$ has a limiting distribution which is $N(0,1)$ as $n \to \infty$.

8. (12 pts.) Let $X_1, ..., X_n$ be a r.s. from a $N(\mu, \sigma^2)$ distribution and let $c_1, ..., c_n$ be real numbers. Identify the distributions of the following r.v.'s (it is not necessary to provide the details).

a). $\sum_{i=1}^{n} c_i X_i$

b). $\overline{X}$

c). $\sum_{i=1}^{n} \frac{(X_i - \mu)^2}{\sigma^2}$

d). $\sum_{i=1}^{n} \frac{(X_i - \overline{X})^2}{\sigma^2}$