

American University of Beirut

STAT 230

Introduction to Probability and Random Variables

Fall 2007

Final Exam

Name:

ID #:

Exercise 1 Let X_1, X_2, X_3 be three independent random variables with binomial distributions $b(4, 1/2)$, $b(6, 1/3)$, and $b(12, 1/6)$ respectively.

- a. find $P(X_1 = 2, X_2 = 2, X_3 = 5)$
- b. find $E(X_1 X_2 X_3)$
- c. find the mean and the variance of $Y = X_1 + X_2$

Exercise 2 The joint pdf of two random variables X and Y is

$$f(x, y) = kx \quad 0 < x < y < 1$$

- a. find the value of the constant k
- b. find the marginal pdf of X and Y . Are they independent?
- c. find $P(X + Y < 1/2)$
- d. find $E(X^2 Y)$

Exercise 3 A man and a woman decide to meet at a certain location. If each person independently arrives at a time uniformly distributed between 12 noon and 1 PM, find the probability that the first to arrive has to wait no longer than 10 minutes.

Exercise 4 Let X and Y be a random sample of from the exponential distribution with pdf

$$f(x) = e^{-x} \quad 0 < x < \infty$$

Let $U = X/Y$ and $V = X + Y$. Find the joint pdf of the couple (U, V) . Are U and V independent?

Exercise 5 Let X_1, X_2, \dots, X_n be a random sample of size n with uniform distribution over $(0, 1)$. Find the pdf of $Y = \max(X_1, X_2, \dots, X_n)$. Find $E(Y)$, $Var(Y)$ and $M_Y(t)$, the moment generating function of Y .

good luck