

American University of Beirut
STAT 230
Introduction to Probability and Random Variables
Spring 2008

Final Exam

Name:

ID #:

circle your section please: 1 (TTH 8 am) 2 (TTH 2 pm)

Exercise 1 (20 points) Let $f(x) = \frac{2}{5}|x|$, if $-1 < x < 2$, be the pdf of a continuous random variable X . Find the pdf of $Y = X^2$.

Exercise 2 (20 points) The joint pdf of two random variables X and Y is

$$f(x, y) = ky \quad 0 < x < y < 1$$

- a. find the value of the constant k
- b. find the marginal pdf of X and Y . Are they independent?
- c. find $P(XY < 1/2)$
- d. find $E(X^2Y)$

Exercise 3 (20 points) Let X_1, X_2 be two independent chi-square random variables with r_1 and r_2 degrees of freedom, respectively. Let $Y_1 = (X_1/r_1)/(X_2/r_2)$ and $Y_2 = X_2$. Find the joint pdf of Y_1, Y_2 , then find the marginal pdf of Y_1 .

Exercise 4 (25 points) Let X_1, X_2, X_3 denote a random sample of size $n = 3$ from a distribution with the geometric pdf

$$P(X = k) = \left(\frac{3}{4}\right) \left(\frac{1}{4}\right)^{k-1} \quad k = 1, 2, 3, \dots$$

- a. find $P(X_1 = 1, X_2 = 3, X_3 = 1)$
- b. determine $P(X_1 + X_2 + X_3 = 5)$
- c. find the moment generating function of $Z = X_1 + X_2 + X_3$, then give the distribution of Z
- d. let $Y = \max(X_1, X_2, X_3)$
 - i. find $P(Y \leq 2)$
 - ii. find $P(Y \leq n)$, then deduce $P(Y = n)$

Exercise 5 (15 points) In the casino game roulette, the probability of winning with a bet on red is $p = 18/38$. Let Y equal the number of winning bets out of 1000 independent bets that are placed. Approximate $P(Y > 500)$.

(hint: consider $Y = X_1 + \dots + X_{1000}$ where X_i is the winning at the i th bet, then use the central limit theorem)

good luck