American University of Beirut

STAT 230

Introduction to Probability and Random Variables

Fall 2007

Final Exam

Exercise 1 *(15 points)* Let $f(x) = \frac{2}{5}|x|$, if $-1 < x < 2$, be the pdf of a continuous random variable X. Find the cdf and the pdf of $Y = X^2$.

Exercise 2 Let X be a continuous random variable with pdf

$$f(x) = kx^2 e^{-x^2/2} \qquad 0 < x < +\infty$$

a. *(7 points)* find the value of the constant k

b. *(8 points)* find $E(X)$ and $Var(X)$

Exercise 3 *(10 points)* Show that probability that the fifth head is observed on the tenth independent flips of a fair coin is 63/512.

Exercise 4 *(10 points)* Roll a pair of fair dice. Let X denote the maximum of the two faces and Y the minimum of the two faces. Compute $Cov(X, Y)$.

Exercise 5 Let X_1 and X_2 be independent $\chi^2(2)$ distributions, i.e.

$$f(x) = \frac{1}{2}\, e^{-x/2} \qquad 0 < x < +\infty$$

a. *(10 points)* find the joint pdf of $Y_1 = X_1 + X_2$ and $Y_2 = X_1 - X_2$

b. *(10 points)* find the marginal pdf of Y_2

Exercise 6 *(15 points)* Let X and Y be two independent Binomial random variables with parameters n and $p = 1/2$. Show that $P(X = Y) = \dfrac{(2n)!}{(n!)^2.2^{2n}}$

hint: you may use the identity $\displaystyle\sum_{k=0}^{n} \binom{n}{k}^2 = \binom{2n}{n}$

Exercise 7 *(15 points)* On each bet, a gambler loses 1 with probability 0.7, loses 2 with probability 0.2, or wins 10 with probability 0.1. Approximate the probability that the gambler will be losing after the first 100 bets.
(*hint: you may use the Central Limit Theorem, and that* $F_Z(-0.29) = 0.39$)

good luck