American University of Beirut

STAT 230

Introduction to Probability and Random Variables

Summer 2006

quiz # 2

Exercise 1 *(6 points)* Let X be a random a random variable with pdf

$$f(x) = \begin{cases} 1/3 & -1 < x < 0 \\ \frac{4}{15}(x+1) & 1 < x < 2 \end{cases}$$

a. Find the cdf of X

b. Find $P(X < -1/2)$, and $P(-1/4 < X < 3/2)$

c. Find $M_X(s)$, the moment generating function of X

d. Find $E(X)$ and $Var(X)$

Exercise 2 *(5 points)* Let X be a random variable with uniform distribution on $(0, 1)$, i.e. the pdf of X is

$$f(x) = 1 \qquad 0 < x < 1$$

Let $Y = -2 \ln X$. Find the cdf of Y, and use it to deduce the pdf of Y.

Exercise 3 *(4 points)* Let X and Y be two random variables with joint pdf

$$f(x, y) = \frac{k}{y} \qquad x > 0\ ,\ y > 0\ ,\ y > x\ ,\ y < 1/x$$

a. Find the value of k

b. Find the marginal distributions of X and Y. Are they independent?

Exercise 4 *(bonus: 3 points)* Let $X \rightsquigarrow \mathcal{N}(1, 3)$ and $Y \rightsquigarrow \mathcal{N}(1, 4)$ be two independent normal distributions. What's the distribution of $X - Y$?