American University of Beirut
STAT 230
Introduction to Probability and Random Variables
Summer 2006

quiz # 3

Exercise 1 There are 5 black chips and 3 white chips in a bowl. The black chips are numbered 1,1,1,2,2, respectively, and the white chips are numbered 1,2,2, respectively. 2 chips are drawn at random and without replacement from the urn. Let X be the number of black balls among the 2, and Y be the sum of the two numbers on the balls.

a. What are the values taken by the couple (X, Y)?

b. Find $P(X = 0 \cap Y = 3)$.

c. Find the joint pdf of X and Y *(give the pdf in rectangular table)*. Are X and Y independent?

d. Find $P(XY = 4)$, and $E(X^2 Y)$.

Exercise 2 Let X and Y have joint pdf

$$f(x, y) = kxy \qquad 0 < x < 1, \ 0 < y < 1$$

a. find the value of the constant k.

b. find $P(XY \leq 1/2)$.

c. find the marginal distributions of X and Y. Are they independent?

d. find the joint pdf of $U = XY$ and $V = X/Y$. Are U and V independent?

e. find $E\left(\dfrac{1}{V}\right)$