American University of Beirut<br>STAT 230<br>Introduction to Probability and Random Variables<br>Summer 2007<br>quiz \# 1

Exercise 1 a. ( 7 points) A cafe lets you order a deli sandwich your way. There are 6 choices for bread, 4 choices for meat, 4 choices for cheese, and 12 different garnishes. How many different sandwich possibilities are there if you choose one bread; 0,1 , or 2 meats; 0,1 , or 2 cheeses; and from 0 to 12 garnishes ?
b. (5 points) Among nine presidential candidates at a debate, three are Republicans and six are Democrats. How many lineups by party are possible if each candidate is labeled either R or D ?
c. (8 points) Let $A, B$ and $C$ be three mutually independent events. Show that the events $A$ and $(B \cup C)$ are independent.

Exercise 2 In a string of 12 Christmas tree light bulbs, 3 are defective. The bulbs are selected at random and tested, one at a time, until the third defective bulb is found.
a. ( 7 points) if the draw is without replacement, find the probability that the third defective bulb is the tenth selected.
b. (8 points) if the draw is with replacement, what is the expected number of draws needed?

Exercise 3 (10 points) A purchaser of electrical components buys them in lots of size 10. It is his policy to inspect 3 components randomly from a lot and accept the lot only if all 3 are non defective. If 30 percent of the lots have 4 defective components and 70 percent have only 1 , what proportion of lots does the purchaser reject?

Exercise 4 (10 points) If independent trials, each resulting in a success with probability $2 / 3$, are performed, find the probability of 8 successes occurring before 5 failures.

Exercise 5 (15 points) Person A tosses a coin and then person B rolls a die. This is repeated independently until a head or one of the numbers $1,2,3,4$ appears, at which time the game is stopped. Person A wins with the head and B wins with one of the numbers 1,2,3,4. Compute the probability that A wins the game

Exercise 6 (10 points) There is a new diagnostic test for a disease that occurs in about $0.05 \%$ of the population. The test is not perfect but will detect a person with the disease $99 \%$ of the time. It will, however, say that a person without the disease has the disease about $3 \%$ of the time. A person is selected at random from the population and the test indicates that this person has the disease. Find the conditional probability that the person does not have the disease.

