American University of Beirut STAT 230

Introduction to Probability and Random Variables
Summer 2007

quiz # 1

- Exercise 1 a. (7 points) A cafe lets you order a deli sandwich your way. There are 6 choices for bread, 4 choices for meat, 4 choices for cheese, and 12 different garnishes. How many different sandwich possibilities are there if you choose one bread; 0, 1, or 2 meats; 0, 1, or 2 cheeses; and from 0 to 12 garnishes?
- **b.** (5 points) Among nine presidential candidates at a debate, three are Republicans and six are Democrats. How many lineups by party are possible if each candidate is labeled either R or D?
- **c.** (8 points) Let A, B and C be three mutually independent events. Show that the events A and $(B \cup C)$ are independent.

Exercise 2 In a string of 12 Christmas tree light bulbs, 3 are defective. The bulbs are selected at random and tested, one at a time, until the third defective bulb is found.

- **a.** (7 points) if the draw is **without replacement**, find the probability that the third defective bulb is the tenth selected.
- **b.** (8 points) if the draw is **with replacement**, what is the expected number of draws needed?

Exercise 3 (10 points) A purchaser of electrical components buys them in lots of size 10. It is his policy to inspect 3 components randomly from a lot and accept the lot only if all 3 are non defective. If 30 percent of the lots have 4 defective components and 70 percent have only 1, what proportion of lots does the purchaser reject?

Exercise 4 (10 points) If independent trials, each resulting in a success with probability 2/3, are performed, find the probability of 8 successes occurring before 5 failures.

Exercise 5 (15 points) Person A tosses a coin and then person B rolls a die. This is repeated independently until a head or one of the numbers 1,2,3,4 appears, at which time the game is stopped. Person A wins with the head and B wins with one of the numbers 1,2,3,4. Compute the probability that A wins the game

Exercise 6 (10 points) There is a new diagnostic test for a disease that occurs in about 0.05% of the population. The test is not perfect but will detect a person with the disease 99% of the time. It will, however, say that a person without the disease has the disease about 3% of the time. A person is selected at random from the population and the test indicates that this person has the disease. Find the conditional probability that the person does not have the disease.