American University of Beirut

STAT 230

Introduction to Probability and Random Variables

Fall 2007-2008

quiz # 2

Name: ..... ID #: .....

Exercise 1 *(25 points)* Let X be a random a random variable with pdf

$$f(x) = \begin{cases} 1/2 & 0 < x < 1 \\ 1/2 & 2 < x < 3 \\ 0 & elsewhere \end{cases}$$

a. find the cdf of X

b. find $P(1/3 < X \leq 7/3)$

c. find $E(X)$ and $Var(X)$

d. find $M_X(t)$, the moment generating function of X

Exercise 2 *(20 points)* The life X in years of a voltage of a car has the pdf

$$f(x) = \frac{3x^2}{7^3}\, e^{-(x/7)^3} \qquad 0 < x < +\infty$$

a. find the probability that the regulator will last at least 7 years.

b. given that it has lasted at least 7 years, find the probability that it will last at least another 3.5 years.

Exercise 3 *(25 points)* Let X be a random variable with pdf

$$f(x) = \frac{e^{-x}}{(1+e^{-x})^2} \qquad -\infty < x < +\infty$$

Show that $Y = \dfrac{1}{1+e^{-X}}$ have a uniform distribution $\mathcal{U}(0,1)$

Exercise 4 *(30 points)* Let X be a random variable with pdf

$$f(x) = a\, e^{-|x|} \qquad -\infty < x < +\infty$$

a. find the value of the constant a

b. show that $E(X^n) = \frac{1}{2}\,(1+(-1)^n).n!$

c. let $Z = |X|$. Find the pdf of Z