

American University of Beirut

STAT 230

Introduction to Probability and Random Variables

Fall 2007

quiz # 1

Exercise 1 (*4 points*)

- a. In a lot of 10 light bulbs, there are 2 bad bulbs. An inspector examines 5 bulbs, which are selected at random and without replacement. Find the probability of at least one defective bulb among the 5.
- b. If $P(C_1) > 0$ and if C_2, C_3, C_4, \dots are mutually disjoint sets, show that $P(C_2 \cup C_3 \cup \dots | C_1) = P(C_2 | C_1) + P(C_3 | C_1) + \dots$
- c. If n fair dice are rolled, find the probability that exactly two of them show up 6.

Exercise 2 (*6 points*) Sam and Jad are involved in a duel. The rules of the duel are that they are to pick up their guns and shoot at each other simultaneously. If one or both are hit, then the duel is over. If both shots miss, then they repeat this process. Suppose that the result of the shots are independent and that each shot of Sam will hit Jad with probability $1/2$, and each shot of Jad will hit Sam with probability $1/3$.

- a. Find the probability that the duel ends at the third round of shots.
- b. Find the probability that Sam wins the duel.

Exercise 3 (*5 points*) Bowl I contains 6 red chips and 4 blue chips. Five of these 10 chips are selected at random and without replacement and put in bowl II, which was originally empty. One chip is then drawn at random from bowl II.

- a. What's the probability that this chip is blue?
- b. Given that this chip is blue, find the conditional probability that 2 red chips and 3 blue chips are transferred from bowl I to bowl II.

Exercise 4 (*5 points*) Balls are drawn at random, one at a time and without replacement from an urn that contains 3 red balls and 3 white balls. Let X be the number of draws needed until only those of the same color are left. Find the pdf of X . Find the mgf of X , $E(X)$, and $Var(X)$.