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II 10
III 20
IV 10
V 20
VI 17
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AMERICAN UNIV. of BEIRUT

QUIZ - I - M.204. Nov. 9,2002

Name: [redacted] ….. Sec: 1 2 3 4 5 6 7 8 9 10

*Instructions:*

1. Write your name and circle your section number.
2 Answer,in detail, every part in the space provided for it ,and circle your final result .Do not mention just the answer.
3.The colored booklet is for scratch work and will not be corrected.

$X = A^{-1} B$.

$2x_1 + 3x = 4$.
$5x_1 + 6x_2 = 8$.

I.The solution to a system of equations having the form $AX = B$ can be found by the matrix multiplication : $X = \begin{pmatrix} 7 & -1 \\ 12 & -6 \end{pmatrix}\begin{pmatrix} 60 \\ -24 \end{pmatrix}$ which is $X = A^{-1} B$

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1.(10%) What is the original system of equations?

$AX = B$ $A^{-1}$ is $\begin{pmatrix} 7 & -1 \\ 12 & -6 \end{pmatrix}$ ⇒ we bring the inverse of this matrix call the inverse M

M of $A^{-1} = \frac{1}{|\Delta|} A_c^{-1 t}$ ⇒ $\Delta = -42 + 12 = -30$ and $A_c^{-1} = \begin{pmatrix} -6 & -12 \\ 1 & 7 \end{pmatrix}$ and its transpose is $\begin{pmatrix} -6 & 1 \\ -12 & 7 \end{pmatrix}$ ⇒ $M = \begin{pmatrix} \frac{1}{5} & \frac{1}{30} \\ \frac{2}{5} & \frac{7}{30} \end{pmatrix}$ original system $AX = B$

$\frac{1}{5}x_1 - \frac{1}{30}x_2 = 60$
$\frac{2}{5}x_1 - \frac{7}{30}x_2 = -24$

2.(5%) What is the solution ?

$X = \begin{pmatrix} 7 & -1 \\ 12 & -6 \end{pmatrix}\begin{pmatrix} 60 \\ -24 \end{pmatrix}$

$\begin{pmatrix} 420 + 24 \\ 720 + 144 \end{pmatrix} = \begin{pmatrix} 444 \\ 864 \end{pmatrix}$ [illegible]

$x_1 = 444$
$x_2 = 864$

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II.(10%)The following matrix gives the transition probabilities related to a market dominated by two firms, A and B:

$$T = \begin{pmatrix} 0.4 & 0.6 \\ 0.2 & 0.8 \end{pmatrix}$$

Assume currently firm A has 65% of the market share.
1.(5%) Predict the market share of A in the next period.

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$T = \begin{pmatrix} 0.4 & 0.6 \\ 0.2 & 0.8 \end{pmatrix}$ $S = (0.65 \quad 0.35)$

share for next period $= ST = (0.65 \quad 0.35)\begin{pmatrix} 0.4 & 0.6 \\ 0.2 & 0.8 \end{pmatrix} = (0.33 \quad 0.67)$

1×2 2×2 1×2 [illegible]

2.~~(10%)~~ Assume the transition matrix remains stable. Find the expected equilibrium shares of the two firms, if they exist.

• Equilibrium: $ET = E$

$E = (P_1 \quad P_2)$ $P_1 + P_2 = 1$ [illegible]

$(P_1 \quad P_2)\begin{pmatrix} 0.4 & 0.6 \\ 0.2 & 0.8 \end{pmatrix} = (P_1 \quad P_2)$

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$0.4P_1 + 0.2P_2 = P_1$ $0.4(1 - P_2)$ [illegible] $1 - P_2$
$0.6P_1 + 0.8P_2 = P_2$ $0.8P_2$ [illegible] $P_2 = 0.75$

III.(20%)Given the matrix $A=\begin{pmatrix}-5 & 1 & -2\\ -4 & 0 & -2\\ 3 & -2 & -3\end{pmatrix}$

1.Calculate $|A|$ by adding columns. Show your work.

$$|A| = \begin{vmatrix}-5 & 1 & -2\\ -4 & 0 & -2\\ 3 & -2 & -3\end{vmatrix}\begin{matrix}-5 & 1\\ -4 & 0\\ 3 & -2\end{matrix} = (0-6-16)-(0-20+12)$$
$$-22+8=\boxed{-14}$$

2. Use Cramer's rule to solve just for $x_3$ in the system: $A\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix}=\begin{pmatrix}28\\ -14\\ -7\end{pmatrix}$. Here $A$ is the given matrix.

Cramers Rule says : $X_3 = \frac{|\Delta x_3|}{|A|}$

and $|A|$ we brought in the point before $= -14$

and $\Delta x_3$ : $\begin{vmatrix}-5 & 1 & 28\\ -4 & 0 & -14\\ 3 & -2 & -7\end{vmatrix}\begin{matrix}-5 & 1\\ -4 & 0\\ 3 & -2\end{matrix} = (0-42+224)-(0-140+28)$

$(182+112=\boxed{294}$

$\Rightarrow X_3 = \frac{\Delta x_3}{\Delta A} = \frac{294}{-14} = \boxed{-21}$

3. Find just the 3rd column of $A_c$ (the matrix of cofactors of $A$).

$A=\begin{pmatrix}-5 & 1 & -2\\ -4 & 0 & -2\\ 3 & -2 & -3\end{pmatrix}$

- cofactor of $-2 = \boxed{8}$
- " of $-2 = -(10-3) = \boxed{-7}$ $\Rightarrow$
- " of $-3 = (0+4) = \boxed{4}$

3rd column of $A_c$ is $\begin{pmatrix}8\\ -7\\ 4\end{pmatrix}$

4.Determine $A^2 = A\times A$

$$\begin{pmatrix}-5 & 1 & -2\\ -4 & 0 & -2\\ 3 & -2 & -3\end{pmatrix}\begin{pmatrix}-5 & 1 & -2\\ -4 & 0 & -2\\ 3 & -2 & -3\end{pmatrix} =$$

$(25-4-6)$ $(-5+0+4)$ $(10-2+6)$ sample of how we bring it

$$= \begin{pmatrix}15 & -1 & 14\\ 14 & 0 & 14\\ -16 & 9 & 7\end{pmatrix}$$

IV.(10%) Find the 3×3 matrix B for which $b_{ij} = \begin{cases} j-2i+1 & ; \quad if \quad i=j \\ 2i^2 \cdot j & ; \quad if \quad i \neq j \end{cases}$

$i = j$ then $j - 2i + 1$

$a_{11}$ then $1 - 2 + 1 = 0$

$a_{12} \Rightarrow 2i^2 \cdot j = 2(1)(2) = 4$

$a_{13} = 2i^2 \cdot j = 2(1)(3) = 6$

$a_{21} \Rightarrow 2(2)^2(1) = 8$

$a_{22} \Rightarrow j - 2i + 1 = 2 - 4 + 1 = -1$

$a_{23} = 2(4)(3) = 24$

$a_{31} = 2(9) = 18$ and $a_{32} = 2(9)(2) = 36$ and $a_{33} = 3 - 2(3) + 1 = -2$

$$\begin{pmatrix} 0 & 4 & 6 \\ 8 & -1 & 24 \\ 18 & 36 & -2 \end{pmatrix}$$

V.(20%)The table below has been provided by the quality control department of a factory:

| | Product 1 ($P_1$) | Product 2 ($P_2$) | Product 3 ($P_3$) | Total |
|---|---|---|---|---|
| (A)Acceptable | 50 | 42 | 78 | 170 |
| (B)Unacceptable | 12 | 8 | 10 | 30 |
| Total | 62 | 50 | 88 | 200 |

One item is randomly taken from the batch. Find the following probabilities:

1. P(A)

$P(A) = \frac{170}{200} = \frac{17}{20} = \boxed{0.85}$

2. $P(A \cap P_1)$

$= \frac{50}{200} = \frac{5}{20} = \frac{1}{4} = \boxed{0.25}$

3. $P(B \cup P_2)$

$= P(B) + P(P_2) - P(B \cap P_2)$

$= \frac{30}{200} + \frac{50}{200} - \frac{8}{200} = \frac{72}{200} = \boxed{0.36}$

4. $P(P_3 | A)$

$= \frac{P(P_3 \cap A)}{P(A)} = \frac{78/200}{170/200} = \frac{78}{170} = \boxed{0.45}$