Version 1
SOLUTIONS

Name: **ID number:**

Time: 1 hour | Math 204 | 8/04/06

Quiz II

Second Semester 05/06

Instructor: Mrs. Muna Jurdak

Section 1 Thurs. 11:00 a.m. Section 2: Fri. 1:00 p.m.

Section 3: Thurs. 3:30 p.m. Section 4: Thurs. 2:00 p.m.

Instructions:

1. Write your name and ID number clearly where indicated.
2. Circle your section number above, according to the time of the problem solving session in which you are enrolled.
3. **Solve the problems on this, the white question sheet**. Use the colored sheets for scratch work only. You may use the back of a white sheet to complete the solution of a problem.
4. Give numerical answers correct to 2 decimal places.
5. **Show your work** in all the problems.

1	2	3	4	5	6	7	8	Total
/12	/10	/10	/15	/8	/17	/10	/18	/100

(12%) 1. (a)Find $\frac{dy}{dx}$ for $y = (e^{x^2})\ln(\sqrt{x}+1)$ s $= u \cdot v \Rightarrow \frac{dy}{dx} = u'v + uv'$

$$\frac{dy}{dx} = 2xe^{x^2}(\ln\sqrt{x}+1) + e^{x^2}\cdot\frac{1}{2\sqrt{x}}\cdot\frac{1}{\sqrt{x}+1} \quad \left(\text{since } \frac{d x^{1/2}}{dx} = \frac{1}{2}x^{-1/2}\right).$$

(b) Find $f'(1)$ for $f(x) = \frac{10+x}{(x^3+1)^3}$.

$$f'(x) = \frac{(x^3+1)^3\cdot 1 - (10+x)\times 3(x^3+1)^2 \times 3x^2}{(x^3+1)^6}$$

since $f'(1)$ is required, there is no need to simplify $f'(x)$. Just subsditute $x=1$.

$$f'(1) = \frac{2^3 - 11(3)2^2(3)}{2^6} = \frac{8-396}{64} = \frac{-388}{64}$$

(c) Use the chain rule to find $\frac{dy}{dx}$, if $y = \frac{u-1}{u+1}$, and $u = (x^2 - 3x + 1)^{1/3}$.

$$\frac{dy}{du} = \frac{(u+1).1-(u-1).1}{(u+1)^2} = \frac{2}{(u+1)^2}; \quad \frac{du}{dx} = \frac{1}{3}(x^2-3x+1)^{-2/3}(2x-3).$$

$$\frac{dy}{dx} = \frac{2}{(u+1)^2} \times \frac{(2x-3)}{3(x^2-3x+1)^{2/3}} = \frac{2}{[(x^2-3x+1)^{1/3}+1]^2} \cdot \frac{(2x-3)}{3(x^2-3x+1)^{2/3}}$$

(10%) 2. (a) Find $\int(\sqrt{5x} + \frac{3}{\sqrt{x}} - 21x^6 - 1)dx$. $= \int(\sqrt{5}\, x^{1/2} + 3x^{-1/2} - 21x^6 - 1)dx$

$$= \frac{\sqrt{5}\, x^{3/2}}{3/2} + \frac{3x^{1/2}}{1/2} - \frac{21x^7}{7} - x + C = \frac{2}{3}\sqrt{5}\, x^{3/2} + 6\sqrt{x} - 3x^7 - x + C,$$

where C is a constant.

(b) If $g''(x) = \frac{-2}{x^3} + 2$, $g'(1) = 5$ and $g(1) = 9$, find $g(x)$.

We must integrate $g''(x)$ twice to find $g(x)$

$g''(x) = -2x^{-3} + 2 \Rightarrow g'(x) = -\frac{2x^{-2}}{-2} + 2x + C_1 = \frac{1}{x^2} + 2x + C_1$ But.

$g'(1) = 5$ so $g'(1) = \frac{1}{1^2} + 2\times 1 + C_1 = 5 \Rightarrow C_1 = 2$

$\Rightarrow g'(x) = x^{-2} + 2x + 2 \Rightarrow g(x) = \int(x^{-2} + 2x + 2)dx = \frac{x^{-1}}{-1} + x^2 + 2x + C_2$.

$g(x) = -\frac{1}{x} + x^2 + 2x + C_2$. But $g(1) = 9$ so $g(1) = -\frac{1}{1} + 1 + 2 + C_2 = 9$.

$\Rightarrow C_2 = 7$

$\Rightarrow g(x) = -\frac{1}{x} + x^2 + 2x + 7$

(10%) 3. The frequency distribution for a random variable X is given below.

X	1	2	3	6
Frequency	40	20	60	80

(a) Construct the probability distribution of X.

Simply divide frequency of each value of X by total sum of frequencies, which is 200.

X	1	2	3	6
$P(x)$	$\frac{40}{200} = 0.2$	0.1	0.3	0.4

(b)Find p(X ≥ 2). $= p(X=2)+p(X=3)+p(X=6)$

$= 0.1+0.3+0.4 = 0.8$

(c)Find p(X > 6). X cannot be bigger than 6, so $X>6$ is an impossible event $\Rightarrow$
$P(X>6)=0$ (not ϕ : ϕ isn't a number, but the empty set.)

(15%) 4. The time it takes a college freshman to complete a certain reasoning test is normally distributed, with a mean of 25 minutes and standard deviation equal to 6 minutes.

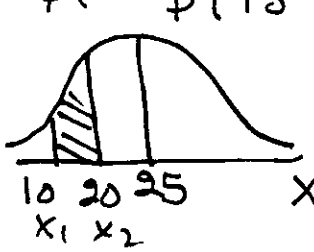
(a)Find the probability that, a student chosen at random, will take between 10 and 20 minutes to finish the test.

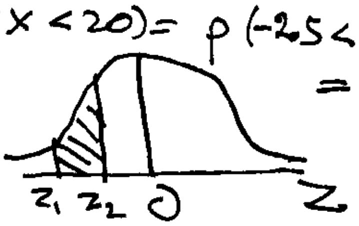
For $x_1=10$ + $x_2=20$, $z_1=\frac{10-25}{6}=-2.5$ $z_2=\frac{20-25}{6}=-0.83$.

p($p(10<X<20)=p(-2.5<z<-0.83)=p(0.83<z<2.5)$

$= 0.4938-0.2967=0.1971$

(from table) ≈ 0.20.





(b) Find the probability that, a student, chosen at random, will take between 5 and 45 minutes to finish the test.

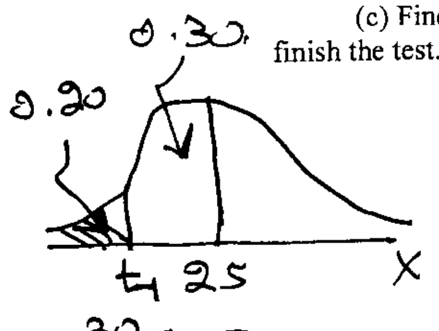
$x_1=5 \Rightarrow z_1=\frac{5-25}{6}=\frac{-20}{6}=-3.33$, $x_2=45 \Rightarrow z_2=\frac{45-25}{6}=3.33$.

$p(-3.33<z<3.33) = 2p(0<z<3.33)$. In the table

$p(0<z<3.09) = 0.4990 \approx 0.50$ (to 2 dec. places).

largest z value $\Rightarrow$ For any larger z, the area is 0.5 $p(-3.33<z<3.33)=2\times0.5=1$

(c) Find the time t_1 below which only 20% of the students take to finish the test.



Left tail is 0.20 $\Rightarrow$ Area between t_1 & mean is 0.30 find z_1 such that $p(z_1<z<0)=0.30$ + z_1 to left of 0.

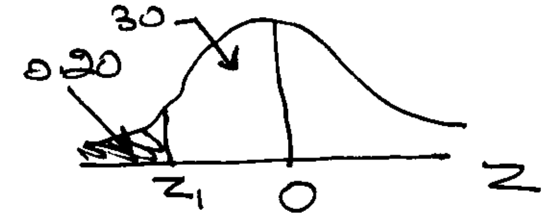
From table, $z_1=-0.84$.

so to find t_1:

$\frac{t_1-25}{6}=-0.84$

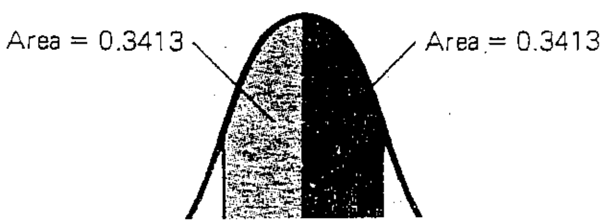
$t_1=(-0.84)\times6+25$

$=19.96$ minutes.



Area Under the Standard Normal Curve

	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2518	.2549
0.7	.2580	.2612	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.49865	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990



(8%) 5. A box contains 4 red balls and 6 white balls. An experiment consists of drawing 5 balls at random from the box, with replacement. Find the probability that at least 3 of the balls are red. This is a Bernoulli experiment where $n=5$. But if success = getting red ball then n. of successes $k \le 4$ since we only have 4 red balls. $p=\frac{4}{10}$, $q=\frac{6}{10}$.

$$P(k\geq 3) = p(k=3) + p(k=4)$$

$$=\binom{5}{3}(0.4)^3(0.6)^2+\binom{5}{4}(0.4)^4(0.6)^1=0.3072 = \boxed{0.31}$$

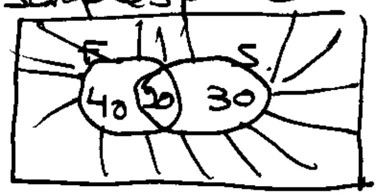
(17%) 6. Of 120 students, 60 are studying French, 50 are studying Spanish, ~~and 20~~ are studying both French and Spanish.

F = event student studying french. S = event he's studying Spanish

(a)A student is chosen at random. Find the probability that:

(i)The student is studying neither French nor Spanish.

Samplespace



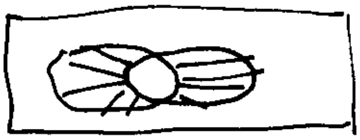
This is the complement of $F\cup S$ (shaded region).

$P(F\cup S) = p(F) + p(S) - p(F\cap S) = \frac{60+50-20}{120} = \frac{90}{120}$.

$1 - p(F\cup S) = 1 - \frac{9}{12} = \frac{3}{12}$.

(ii) The student is studying exactly one of the two languages.

Either French only or Spanish only (shaded region)



$p(F\cap S') + p(S\cap F') = \frac{60-20}{120} + \frac{50-20}{120} = \frac{7}{12}$.

(iii)The student is studying French, given that he is studying Spanish.

$$P(F/S) = \frac{p(F\cap S)}{P(S)} = \frac{2/12}{5/12} = 2/5.$$

(b) Let F be the event that the student is studying French, and S be the event that the student is studying Spanish. Are F and S statistically independent events? Give a clear reason for your answer.

F & S are statistically independent if & only if $P(F/S) = p(F)$. Otherwise they're dependent.

$P(F/S) = \frac{2}{5}$ (from (iii)). $p(F) = \frac{6}{12} = \frac{1}{2}$

$\frac{2}{5} \neq \frac{1}{2} \Rightarrow p(F/S) \neq p(F)$

$\Rightarrow$ F & S are statistically dependent events.

(10%) 7. The scores on a quiz for a class of 40 students have a mean equal to 75 and a standard deviation equal to 9. If the teacher decides to raise each student's grade by 2 grades

(a) What will be the new mean (of the raised grades)? Justify your answer.

Let the old grade be x and the new grade be $x' = x+2$.

$$\overline{x'} = \frac{\sum x'}{40} = \frac{\sum(x+2)}{40} = \frac{\sum x + 2\times 40}{40}$$

$$= \frac{\sum x}{40} + 2 = \bar{x} + 2 = 75+2 = 77$$

(b) What will be the new standard deviation? Justify your answer.

Let the new standard deviation be σ'.

$\sigma' = \sqrt{\dfrac{\sum (x'-77)^2}{40}}$ but each $x' = x+2$,

so each deviation from the mean $x'-77 = x-75$ since both x & the mean increased by 2.

$\Rightarrow \sigma' = \sigma$ & $\sigma' = 9$.

(18%) 8. A pair of dice, one red, one green, is rolled once. The random variable X is the maximum of the 2 numbers obtained on the faces of the dice.

(a)If A is the event that X = 3, list the set of outcomes belonging to this event.

$X = 3$: $\{(3,3), (3,2), (2,3), (3,1), (1,3)\}$

(b) Find the probability that $X \leq 3$.

$X = 1$: $\{(1,1)\}$ $X = 2$: $\{(1,2), (2,1), (2,2)\}$

$X = 3$: As in (a).

So $P(X \leq 3) = \dfrac{\text{number of outcomes for } X \leq 3}{\text{total number of outcomes (36)}}$

$$= \frac{1+3+5}{36} = \frac{1}{4}$$

(c) Find the probability that $X \neq 3$.

$P(X=3) = \frac{5}{36}$, $P(X \neq 3) = 1 - \frac{5}{36} = \frac{31}{36}$

(d) Let A be the event that X = 3, B be the event that X is even, and C be the event that $X \geq 4$.

(i)Are the events A, B and C mutually exclusive? Justify your answer.

2, 4, 6 belong to B.
4, 5, 6 " " C.

$B \cap C = \{$event that $X = 4$ or $6\}$

so $B \cap C \neq \emptyset \Rightarrow$ B & C aren't mutually exclusive $\Rightarrow$
A, B, C are not either.

(ii) Are the events A, B and C collectively exhaustive? Justify your answer.

$A \cup B \cup C = \{$event that $X = 2$ or 3 or 4 or 5 or $6\}$

$X = 1 \notin A \cup B \cup C$. 1 is in sample space, so A, B, C aren't collectively exhaustive.