Physics 211L Final exam
Name: $\qquad$
Section number: $\qquad$
Instructor's Name: $\qquad$
ID number: $\qquad$ DO NOT START THE EXAM BEFORE YOU ARE TOLD TO BEGIN

Grading

| I |  |
| :---: | :--- |
| II-1 |  |
| II-2 |  |
| II-3 |  |
| TOTAL |  |

The duration of this exam is 60 minutes.
No notes or books allowed.
Scientific calculators are allowed
All results should be given with the exact number of significant figures.
I. (55\%) In order to measure the capacitance of an unknown capacitor, the following circuit is connected:


At $\mathrm{t}=0$ the switch is closed, the readings of the voltmeter were recorded every 5 seconds with the following outcome:

| t (seconds) | V(Volts) |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 0 | 12.00 |  |  |  |  |
| 5 | 3.210 |  |  |  |  |
| 10 | 0.920 |  |  |  |  |
| 15 | 0.273 |  |  |  |  |
| 20 | 0.082 |  |  |  |  |
| 25 | 0.019 |  |  |  |  |
|  |  |  |  |  |  |

Note that the resistor has 4\% tolerance.
You may find the formulae on page 4 useful.
a- Derive the relationship between V and t .
b- Choose your variables such that you obtain a linear relationship between them.
c- Use linear regression to find the slope of your line along with its error.
d- Determine the capacitance C along with its error.

## Linear Regression

The method of least squares is used to fit a curve (find a theoretical equation) to a set of experimental data. First assume that a linear relation exists between $y$ and $x$

$$
\begin{equation*}
y=A x+B \tag{1}
\end{equation*}
$$

Substitution of $x=x_{i}$ will in general not give the value of $y_{i}$. The "errors" will be

$$
\begin{equation*}
\mathrm{e}_{\mathrm{i}}=\mathrm{y}-\mathrm{y}_{\mathrm{i}}=\mathrm{Ax} \mathrm{x}_{\mathrm{i}}+\mathrm{B}-\mathrm{y}_{\mathrm{i}} \tag{2}
\end{equation*}
$$

To determine the best straight line that fits the N, sets of data, A and B have to be chosen so that the sum of the squares of the "errors" is minimized. This means that the simultaneous equations, obtained by equating the partial derivatives of $\left(y-y_{i}\right)^{2}$ with respect to $A$ and $B$ to zero, should be solved. This condition leads then to the following results

$$
\begin{equation*}
A=\frac{N \sum\left(x_{i} y_{i}\right)-\sum x_{i} \sum y_{i}}{\Delta} \tag{3}
\end{equation*}
$$

and

$$
\begin{equation*}
B=\frac{\sum x_{i}^{2} \sum y_{i}-\sum x_{i} \sum\left(x_{i} y_{i}\right)}{\Delta} \tag{4}
\end{equation*}
$$

where

$$
\begin{equation*}
\Delta=N \sum x_{i}^{2}-\left(\sum x_{i}\right)^{2} \tag{5}
\end{equation*}
$$

The correlation coefficient $r$ provides an indicator of how good a fit the best straight line is. This coefficient is defined as

$$
\begin{equation*}
r=\frac{\sum\left(x_{i}-\bar{x}\right)\left(y_{i}-\bar{y}\right)}{\sqrt{\sum\left(x_{i}-\bar{x}\right)^{2} \sum\left(y_{i}-\bar{y}\right)^{2}}} \tag{6}
\end{equation*}
$$

For $r=0$, the values of x and y are independent of one another and there is no linear correlation. The closer $r$ is to +1 or to -1 , the better the linear correlation is.
Finally, the error in A is given by:

$$
\sigma_{A}^{2}=\frac{N}{N-2} \frac{\sum e_{i}^{2}}{\Delta}
$$

## II. Questions

1. In the "Electrical Circuits" experiment, how was the resistivity of a copper coil determined? Explain briefly the procedure and include necessary equations. (20\%)
2. (15\%) The graph in the figure below shows the current I in a diode as a function of potential difference $\Delta V$ across the diode.
a- Determine the resistance of the diode for six different values of $\Delta V$ in the range from -1.5 V to 1 V .(5\%)
b- Based on your results, what electrical property does a diode possess? Does this classify the diode as an Ohmic device? Explain.(10\%)

(a)
3. In the "Force between Two Parallel Conductors" experiment, how did we eliminate the effect of the earth magnetic field? (10\%)
