AMERICAN UNIVERSITY OF BEIRUT
Mathematics Department-FAS

MATH 251
QUIZ 1
Fall 2014-2015
Closed Book, 75 mn

STUDENT NAME	
ID NUMBER	

Problem	Out of	Grade
1	11	
2	18	
3	10	
4	11	
TOTAL	50	

1. **(11 points)** Let d be a number given in decimal representation:

$$d = (123.175)_{10} \times 10^{31}$$

To convert d to a binary number in the IEEE Single Precision floating point system, proceed as follows:

(a) (6 pts)Convert first d to an Octal number $(d)_8$.

$(d)_8 =$

(b) (5 pts)Convert then $(d)_8$ to a binary number $(d)_2$ in IEEE Single precision, and write its Hexadecimal representation. (Use rounding to the closest, if needed).

$(d)_2$=

Hexadecimal representation of x:

2. **(18 points)**

(a) (2 pts) Let x be a NEGATIVE binary number with $|x| > x_{max}$ in the IEEE Single precision system. Write the hexadecimal representation $[H_1\ H_2\ H_3\ H_4\ H_5\ H_6\ H_7\ H_8]_{16}$ of x in that case.

(b) (1 pt) Deduce then the hexadecimal representation in Single precision of the binary number $x = -2^{+128}$.

(c) (4 pts)In IEEE Single precision, find the next largest machine number (successor(x)) for the binary number x above. Identify successor(x), then write its Hexadecimal representation. Justify all steps.

$Successor(x)$

- Hexadecimal representation of $successor(x)$ in Single precision:

(d) (4 pts) Let x_M be the midpoint of the interval $[0, x = -2^{+128}]$. Write the machine number representing x_M in IEEE Single Precision, then write its HEXADECIMAL representation:

- machine number representing x_M in IEEE Single Precision

- HEXADECIMAL representation of x_M in Single Precision

(e) (4 pts)In IEEE Single precision, find the next smallest machine number (predecessor(x_M)) for the binary number x_M found above, then write its Hexadecimal representation. Justify all steps.

$Predecessor(x_M)$

- Hexadecimal representation of $predecessor(x_M)$ in Single precision:

(f) (3 pts) In IEEE Double precision: write first x in normalized floating point form, then write its corresponding Hexadecimal representation.

- $x=$

- Hexadecimal representation of x in Double precision

3. **(10 points)** Consider the real valued function

$$f(x) = \frac{e^x - \cos x}{\sin x}$$

(a) (2 pts) For which values of x is there a loss of precision when computing $f(x)$. Justify your answer.

(b) (5 pts) Find a remedy to this problem based on adequate Taylor's series expansion .
Hint: Note that $\lim_{x \to 0} \frac{\sin x}{x} = 0$

(c) (3 pts) Use the first 3 terms of the suggested remedy, to compute $f(10^{-3})$ up to 7 decimal digits (p=7), with rounding to the closest.

4. **(11 points)** Consider the function $f(x) = (2x-1)(x+3) + \frac{1}{x+1}$ defined on $D_f = (-\infty, -1) \cup (-1, +\infty)$.

   (a) (6 pts)**Locate** all the roots of $f(x)$ on D_f, through either graphing $f(x)$ or by using a fixed point argument.

(b) (5 pts)In order to compute the positive root of $f(x)$ up to 4 decimal figures with rounding to the closest in the interval $(0,1)$, apply first the bisection method twice, followed by Newton's method .

- Bisection method twice:

- Newton's method up to 4 decimal figures: