AMERICAN UNIVERSITY OF BEIRUT
Faculty of Arts and Sciences
Mathematics Department

MATH 251/CMPS 251
MID TERM EXAMINATION
FALL 2004-2005

Closed Book, One hour 15 minutes

SUBMIT THE QUESTION SHEET WITH BOOKLET
(ONLY SCIENTIFIC CALCULATORS ARE ALLOWED.
PROGRAMMABLE AND GRAPHIC CALCULATORS ARE FORBIDDEN)

STUDENT NAME	
ID NUMBER	

1. Let $x \in \mathbb{F} \equiv \mathbb{F}(b, p, e_{min}, e_{max})$, with $x = \pm m \times b^e$.

-(5 points) Fill in the bounds on m

$$\ldots\ldots\ldots\ldots\ldots\ldots \leq m \leq \ldots\ldots\ldots\ldots\ldots\ldots$$

and if $m_2 = succ(m_1)$, both m_1 and m_2 mantissas in $\mathbb{F}$, find $m_2 - m_1$.

$$m_2 - m_1 = \ldots\ldots\ldots\ldots\ldots\ldots$$

-(5 points) Assume $b = 2$, $p = 5$, $e_{min} = -3$, $e_{max} = 3$. How many **positive numbers** does $\mathbb{F}$ includes ? -(10 points)Fill in the missing statements in the following MATLAB program that generates the non-negative elements of a floating-point system $\mathbb{F}(b, p, e_{min}, e_{max})$.

```
function x=float(b,p,emin,emax)
x=0;
epsm=b^(-p+1);
%M represents all possible values taken by the mantissa
M=.............................;
E=1;
for ..........................
    x=[x ...................];
    E=E.......................;
end
E=.....................................................................:
for ..........................
    x=[x ................];
    E=E.......................;
end
```

-(5 points) How many floating-point operations (additions and multiplications) would be required to execute the above program.

2. Consider the floating-point system $\mathbb{F} = \mathbb{F}(\ 10, 6, -4, 5)$. This system uses rounding to the closest.

   (a) Fill in the following table. (10 points)

Values of following parameters in $\mathbb{F}$	
$x_{\min}$	
$x_{\max}$	
ϵ_M (epsilon machine)	
Representation of $\frac{1}{11}$	
succ($\frac{1}{11}$)	

   (b) (10 points) Convert $x = -(652.2025)_{10}$ into octal form ? Give then the hexadecimal form of the internal IEEE single precision floating point representation of x, using rounding to the closest.

Conversion of $x = -(652.2025)_{10}$	to octal and IEEE hexadecimal form
Corresponding octal form	
Corresponding IEEE hexadecimal form	

3. Consider the function $f(x) = e^{-x} - \sin(x)$.

   (a) (5 points) How many roots does this function have on $(0, 4)$. Graph this function on this interval and locate the first root r of $f(x)$.

(b) (5 points) Find the least number of iterations that provide an approximation to r within 5 significant figures using the bisection method.

Number of iterations :	

IN WHAT FOLLOWS CARRY ALL YOUR COMPUTATIONS WITH AT LEAST 5 FIGURES

(c) (10 points) Compute the following iterations :
-Give the sequence of 2 approximations obtained by applying 2 iterations of the bisection method with $a = 0$, $b = 1$.

x_1 :	
x_2 :	

-Give the iteration function $r_n = g(r_{n-1})$ of Newton's method :

Then compute the sequence of 2 approximations obtained by applying 2 iterations of Newton's method with $x_0 = 0.5$.

x_1 :	
x_2 :	

-Give the iteration function $r_n = g(r_{n-1}, r_{n-2})$ of the secant method :

Then give the sequence of 2 approximations obtained by applying 2 iterations of the secant method with $x_0 = 1$ and $x_1 = 0.5$.

x_1 :	
x_2 :	

4. Consider finding the cubic root r of a positive number a, $r = a^{1/3}$. Write Newton's process that would yield an approximation to r. Does it converge for any initial choice of the iteration ? Justify by any argument.