

AMERICAN UNIVERSITY OF BEIRUT

Faculty of Arts and Sciences - Mathematics Department.

Test II - MATH 251

Closed Book – 75 mn

SPRING 2007 – 2008

STUDENT NAME _____.

ID NUMBER _____.

Problem 1 _____ / 14 .

Problem 2 _____ / 14 .

Problem 3 _____ / 22 .

TOTAL _____ / 50

TABLE 1: Problems 1 and 2

Consider the following table of data for the function $f(x)$. All computations shall be carried out with 8 significant figures.

i	x_i	y_i
0	0.000	1.0000000
1	0.125	1.1108220
2	0.250	1.1979232
3	0.375	1.2663800
4	0.500	1.3196170
5	0.625	1.3600599
6	0.750	1.3895079
7	0.875	1.4093565
8	1.000	1.4207355

1 - a – Use the Central Difference formula to approximate $f'(0.5)$. Improve this result using Richardson's extrapolation of the 1st and 2nd orders. For that purpose, give the formulae that provide φ_h , φ_h^1 , φ_h^2 , then fill out the table that follows. **(10 points)**

$$\varphi_h(f(x)) =$$

$$\varphi_h^1(f(x)) =$$

$$\varphi_h^2(f(x)) =$$

h	$\varphi_h(\cdot)$	$\varphi_h^1(\cdot)$	$\varphi_h^2(\cdot)$
0.5	.	×	×
0.25	.	.	×
0.125	.	.	×

b -Using table 1, calculate the third derivative $f'''(1.000)$, using the Backward Difference Approximation.

(4 points)

$$f'''(x) =$$

$$f'''(1.000) =$$

2 – a- Using table 1, write Newton’s interpolating polynomial of degree 3, that would **best** approximate $f(0.4)$. Find $f(0.4)$.

(4 points)

b – Find the equation of the quadratic spline $s(x)$, on the interval $[0.125, 0.5]$, then draw its graph. Draw clearly the tangent to the curve at each node. (**Let $z_1 = s'(x_1) = 0$**)

(10 points)

3 - Consider the following 4 by 4 square matrix:

$$A = \begin{pmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 3 & -1 \\ 3 & -3 & 0 & 6 \\ 0 & 2 & 4 & -6 \end{pmatrix}$$

- a- Apply on this matrix Gauss Elimination with the scaled partial pivoting strategy, showing the status of the 4 by 4 matrix after each reduction. (Each pivot row and the corresponding multipliers should be identified and circled). Specify the index vector IV after each reduction. **(10 points)**

Reduction 1:

Modified matrix A including multipliers _____ scales

b- Specify the elements of the matrices P, L and U that satisfy the LU decomposition of the matrix A.

(4 points)

$$P = \begin{pmatrix} \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{pmatrix}$$

$$L = \begin{pmatrix} \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{pmatrix} U = \begin{pmatrix} \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{pmatrix}$$

c- Use the matrices P, L and U, to find the determinant of A.

(2 points)

d- Use the matrices P , L and U , to find the **2nd column (ONLY)** of the inverse of A .
(6 points)