AMERICAN UNIVERSITY OF BEIRUT
Mathematics Department-FAS

MATH 251
TEST 1
FALL 2010-2011
Closed Book, 75 mn

STUDENT NAME	
ID NUMBER	

Problem	Out of	Grade
1	**13**	
2	**12**	
3	**13**	
4	**12**	
TOTAL	**50**	

1. (13 points) Determine the Hexadecimal representation of the decimal number

$$x = -318.724 \times 10^{-24}$$

in F_S(single precision). Use the approximation $10^3 \approx 2^{10}$ and rounding to the closest if necessary.

$(x)_2$= (7 points)

Bit-String Representation of $(x)_2$ in F_S: (4 points)

Hexadecimal representation of x in F_S: (2 points)

2. (12 points)

   (a) - What are the numbers in the IEEE Single precision system F_S immediately to the right or to the left of $x = 2^m$ (i.e respectively the successor of x denoted succ(x), and the predecessor of x denoted pre(x)), in the following cases:

   - $x = 2^m$ with $-126 < m < +127$
     pre(x) = (2 points)

     succ(x) = (2 points)

- $x = 2^m$ with $m = -126$. Identify pre(x), then write its Binary bit string representation in F_S.

  pre(x) = (1.5 points)

  Binary Bit string representation of pre(x) : (1.5 points)

(b) - Determine the values of x for which the following function involves a difficulty. What is it ? What remedy do you propose ?

. (5 points)

$$f(x) = (\sqrt{x+4})^{1/2} - (\sqrt{x})^{1/2}$$

3. (13 points)

(a) Locate the roots of $f(x) = e^{-x} + x^2 - x - 5$, and show the results on a graph. (4 points)

(b) Use Newton's method to approximate the **negative** root of f up to 4 decimal figures. Express all your computed results in $F(10, 5, -20, +20)$; round to the closest if needed. (6 points)

(c) How many iterations are theoretically needed to reach such precision using the Bisection method ? Compare with the number of iterations used in (b), and justify the results. (3 points)

4. (12 points) An $(n \times n)$- tridiagonal matrix A_n is a matrix for which all the elements are zeros, except those on the 3 Diagonals. It is defined as follows:

$$A_n = \begin{bmatrix} a1 & b1 & 0 & 0 & \dots & 0 \\ c1 & a2 & b2 & 0 & \dots & 0 \\ 0 & c2 & a3 & b3 & \dots & 0 \\ \dots. & \dots & \dots & \dots & \dots & \dots \\ \dots. & \dots & \dots & \dots & \dots & \dots \\ \dots. & \dots & \dots & c_{n-2} & a_{n-1} & b_{n-1} \\ 0 & \dots & 0 & 0 & c_{n-1} & a_n \end{bmatrix}$$

(a) Apply the **Naive Gauss** elimination on the following (4×4)-tridiagonal matrix A_4. Determine at each reduction the multipliers and the modified elements of the matrix. (6points)

$$A_4 = \begin{bmatrix} 1 & 5 & 0 & 0 \\ 2 & 2 & 7 & 0 \\ 0 & 1 & 4 & -1 \\ 0 & 0 & -2 & 3 \end{bmatrix}$$

...

(b) Extract the Upper Triangular matrix U_4 obtained at the end of this process: (2 points)

$U_4 =$

(c) If the Naive Gauss reduction is applied on the tridiagonal matrix A_n, deduce from the results obtained in(a) and (b)

- Which elements of A are modified at each reduction: (2 points)

- What is the form of the Upper triangular matrix U_n extracted at the end of the process: (2 points)

  $U_n =$