AMERICAN UNIVERSITY OF BEIRUT
Mathematics Department-FAS

MATH 251
FALL 2010-2011
Quiz 2
Closed Book, 75 mn

STUDENT NAME	
ID NUMBER	

Problem	Out of	Grade
1	15	
2	20	
4	15	
TOTAL	50	

1. (15 points) Let A be the following 4×4 matrix:

$$\begin{bmatrix} 2 & 1 & -1 & 1 \\ 1 & 1 & 0 & 3 \\ -1 & 2 & 3 & -1 \\ 3 & -1 & -1 & 2 \end{bmatrix}$$

(a) (8 points) Apply on this matrix, Gauss Elimination with **the scaled partial pivoting strategy**, showing the status of the matrix after each elimination, i.e. each of the pivot rows and the corresponding multipliers should be identified and circled. Update the index vector at each reduction.

(b) (3 points) Extract the 4 by 4 matrices P, L and U which determine the LU-factorization of A.

- P=

- L=

- U =

(c) (4 points) Use **the LU - decomposition of A** to determine the last column of A^{-1}.

2. (20 points) Consider the following set of data

$$D_3 = \{(0,1); (1,2^1); (2,2^2); (3,2^3)\}$$

that corresponds to the function $f(x) = 2^x$.

(a) (10 points) Based on the set D_3, determine the equations of the Natural Cubic spline function $S(x)$, that approximate the function $f(x) = 2^x$.

....

(b) (5 points) Write the equations of $S'(x)$ that approximate the derivative $f^{'}(x)$ on the interval $[0, 3]$.

(c) (5 points)Use the Cubic spline $S(x)$ and its derivative to approximate $f(0.5)$ and $f'(0.5)$ in $F(10, 5, -20, +20)$ rounding to the closest if needed. Calculate then the relative errors for both approximations.

- $f(0.5) \approx$

- Relative Error:

- $f'(0.5) \approx$

- Relative Error:

3. (15 points) Consider the following set of data:

i	x_i	$f(x_i)$
0	0.000	1.000
1	0.125	1.110
2	0.250	1.197
3	0.375	1.266
4	0.500	1.319

(a) (5 points) Write first the **Forward difference** formula $\phi_h(f(x_i))$ that approximates $f^{'}(x_i)$, then derive the expression of the error series $\epsilon(h) = f^{'}(x_i) - \phi_h(f(x_i))$ in the form:

$$\epsilon(h) = c_1 h^{\alpha_1} + c_2 h^{\alpha_2} + c_3 h^{\alpha_3} + ...,$$

by determining the values of the constants $\{\alpha_1, \alpha_2, \alpha_3, ...\}$.

- $\phi_h(f(x_i)) =$

- $\epsilon(h) =$

(b) (5 points) Based on the Forward difference formula, derive Richardson extrapolation operators of orders 1 and 2 and the order of their error series.

- $\phi_h^1(f(x_i)) =$

  Corresponding Error = O(...........)

- $\phi_h^2(f(x_i)) =$

  Corresponding Error = O(...........)

(c) (5 points) For the purpose of improving the approximation to $f'(0.000)$, fill in the empty slots in the following table adequately, starting with $h_0 = 0.5$.
Express all the results obtained in $F(10, 5, -15, +15)$.

h	$\phi_h(.)$	$\phi_h^1(.)$	$\phi_h^2(.)$
$h_0 = 0.5$			
$h_0/2$			
$h_0/4$			

Best approximation to $f'(0.000)$: