AMERICAN UNIVERSITY OF BEIRUT
Faculty of Arts and Sciences
Mathematics Department

**MATH 251**
**QUIZ II**
**FALL 2009-2010**
Closed Book, 75 MINUTES

**WRITE YOUR ANSWERS ON THE QUESTION SHEET**

| | |
|---|---|
| **STUDENT NAME** | |
| **ID NUMBER** | |

| Problem | Out of | Grade |
|---|---|---|
| 1 | 16 | |
| 2 | 13 | |
| 3 | 13 | |
| 4 | 8 | |
| TOTAL | 50 | |

1. Consider a function $f(x)$ given at 4 distinct data points by the following table:

| $\mathbf{i}$ | $\mathbf{x_i}$ | $\mathbf{y_i}$ |
|---|---|---|
| 0 | 1.0 | 0 |
| 1 | 1.5 | 1.76 |
| 2 | 2.0 | 3.01 |
| 3 | 3.0 | 4.77 |

   (a) Use this table to create a lower triangular matrix whose entries are Neville's interpolating polynomials of all orders.
   Note: Show the details of your calculations in the space left below

| $\mathbf{i}$ | $\mathbf{x_i}$ | $\mathbf{y_i = p_i(x)}$ | .......$\mathbf{p_{i,i+1}(x)}$......... | ......$\mathbf{p_{i,i+1,i+2}(x)}$....... | ...$\mathbf{p_{i,i+1,i+2,i+3}(x)}$.... |
|---|---|---|---|---|---|
| 0 | 1.0 | 0 | . | . | . |
| 1 | 1.5 | 1.7 | . | . | . |
| 2 | 2.0 | 3.0 | . | . | . |
| 3 | 3.0 | 4.7 | . | . | . |

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(b) Approximate $f(1.25)$, using the most suitable **Quadratic** interpolation polynomial .

(c) Based on the EXISTENCE and UNIQUENESS properties of $p_{0123}(x)$, and using the LEAST number of additional parameters, find a polynomial that takes the values shown in the given table, and has at $x = 4$ the value 5.(Note that $p_{0123}(4) = 5.5$)

2. Determine the Natural Quadratic Spline based on the first 3 nodes of the Table given in exercise 1.

3. MATLAB QUESTION : NAIVE GAUSS

4. The objective of this exercise is to set a procedure that finds the LU-decomposition of $A^T$, as the product of a lower unit triangular matrix $L_1$, and an upper triangular matrix $V$, based **only** on the LU-decomposition of the matrix $A$ itself.

   In this view, consider the following LU- decomposition of the matrix A obtained through the Naive Gaussian elimination procedure

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 2 & 4 & 1 \\ -2 & 4 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -2 & 3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & -1 \end{bmatrix} = LU$$

   (a) Determine the matrices $D$ and $U_1$ that factor the matrix $U$ as the product of a diagonal matrix $D = Diag(U)$ and a Unit Upper triangular matrix $U_1$, where:

$$U = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} . & 0 & 0 \\ 0 & . & 0 \\ 0 & 0 & . \end{bmatrix} \begin{bmatrix} 1 & . & . \\ 0 & 1 & . \\ 0 & 0 & 1 \end{bmatrix} = DU_1$$

   (b) Determine then, the lower triangular matrix $M$ that factors $A$ as $A = MU_1$.

$$M = \begin{bmatrix} . & 0 & 0 \\ . & . & 0 \\ . & . & . \end{bmatrix}$$

   <u>Justify the results:</u>

(c) Based on the results above, express $A^T$ as the product $A^T = L_1 V$ where $L_1$ is a unit Lower triangular matrix, and $V$ is an upper triangular matrix.
(Hint: $(XY)^T = Y^T X^T$)

$$L_1 = \begin{bmatrix} 1 & 0 & 0 \\ . & 1 & 0 \\ . & . & 1 \end{bmatrix}$$

$$V = \begin{bmatrix} . & . & . \\ 0 & . & . \\ 0 & 0 & . \end{bmatrix}$$

Justify the results:

(d) Identify **(DO NOT CALCULATE )** the multipliers used at each reduction if the Naive Gauss elimination were applied on the matrix $A^T$.

- Multipliers of Reduction 1:

- Multipliers of Reduction 2: