AMERICAN UNIVERSITY OF BEIRUT
Faculty of Arts and Sciences
Mathematics Department

MATH-CMPS 251
MID TERM EXAMINATION
FALL 2006-2007
Closed Book, One hour 15 minutes

SUBMIT THE QUESTION SHEET WITH BOOKLET
(ONLY NON-PROGRAMMABLE AND NON-GRAPHIC CALCULATORS ARE ALLOWED)

STUDENT NAME	
ID NUMBER	

Problem	Out of	Grade
1	10	
2	10	
3	15	
4	15	
TOTAL	50	

1. Let $x \in \mathbb{F} \equiv \mathbb{F}(b, p, e_{min}, e_{max})$, with $x = \pm m \times b^e$.

   -(4 points) Fill in the missing statements in the following `MATLAB` program that generates the positive elements of a floating-point system $\mathbb{F}(b, p, e_{min}, e_{max})$.

```
function x=float(b,p,emin,emax)
x=[];
epsm=b^(-p+1);
%M represents all possible values taken by the mantissa
M=................................;
E=b^emin;
for ..............................
    x=[x M*E];
    E=E*.........;
end
```

   -(3 points) How many floating-point operations (additions and multiplications) would be required to execute the above program.

   -(3 points) Find the cardinality of $\mathbb{F}$ if $\mathbb{F} = \mathbb{F}(7, 6, -4, 4)$

2. Consider the floating-point system $\mathbb{F} = \mathbb{F}(\ 10, 7, -6, 7)$. This system uses rounding to the closest.

(a) (4 points) Fill in the following table.

Values of following parameters and elements in IEEE single precision system	
$x_{\min}$	
$x_{\max}$	
ϵ_M (epsilon machine)	
Representation of $\frac{1}{13}$	
$\text{succ}(\frac{1}{13})$	

(b) (6 points) Convert $x = (653.325)_{10}$ into octal form? Give then the hexadecimal form of the internal IEEE single precision floating point representation of x, using rounding to the closest.

Conversion of $x = (653.325)_{10}$	to octal and IEEE hexadecimal form
Corresponding octal form	
Corresponding IEEE hexadecimal form	

3. Consider the function $f(x) = e^{-x} - 4x(1-x)$.

   (a) (3 points) Show graphically that this function has only 2 distinct roots on $(-\infty, \infty)$. Locate these roots.

(b) (3 points) Find the least number of iterations that provide an approximation to r within 6 significant figures using the bisection method.

Number of iterations:	

-Apply the bisection method twice to find approximations to EACH of the two rooots.

Approximation to first root :	
Approximation to second root:	

(c) (4 points) Let $f(x) = a - \frac{1}{x}$, $a > 0$
-Write the Newton iteration to find the root r of $f(x)$.

-Give the interval (α, β) such that when $x_0 \in (\alpha, \beta)$ the iteration converges. Justify your answer.

(d) (5 points) Prove that the Newton sequence $\{x_n\}$ verify:

$$\left|x_n - \tfrac{1}{a}\right| = a^n \left|x_0 - \tfrac{1}{a}\right|^{2^n}$$

-Assume that $1 \leq a < 2$ and $|x_0 - \frac{1}{a}| \leq \frac{1}{2}$. How many iterations are needed to get $\frac{1}{a}$ within 6 significant figures.

4. Consider the system of linear equations $Ax = b$, where $A \in \mathbb{R}^{4,4}$, x, $b \in \mathbb{R}^4$, given by:

$$\begin{array}{rcrcrcrcl} 9x_1 & + & 4x_2 & + & 7x_3 & + & 5x_4 & = & 20 \\ 2x_1 & & & + & 7x_3 & + & 8x_4 & = & 9 \\ 6x_1 & + & 8x_2 & + & x_3 & & & = & 15 \\ 4x_1 & + & 5x_2 & + & 4x_3 & + & 3x_4 & = & 12 \end{array}$$

(a) (10 points) Give the augmented matrix $[A|b]$ of this system, then solve the given system (i.e. find x_1,x_2,x_3,x_4) using Gauss reduction with **scaled partial pivoting** followed by backward substitution. Specify all the parameters of the reduction, particularly the scales, the multipliers and the interchange of rows through an index vector IV. Carry out this procedure using whenever necessary **fraction representation** of numbers.

(...Continued...)

(b) (10 points) Find the determinant $det(A)$ of the system matrix A, then specify the elements of the 4 by 4 matrices L and U, which verify

$$B = A(IV, :) = LU$$

Write out this equality, exbiting the elements of B,L and U.