AMERICAN UNIVERSITY OF BEIRUT
Faculty of Arts and Sciences
Mathematics Department

MATH-CMPS 251
QUIZ I
FALL 2007-2008
Closed Book, One hour 15 minutes

SUBMIT THE QUESTION SHEET WITH BOOKLET
(ONLY NON-PROGRAMMABLE AND NON-GRAPHIC CALCULATORS ARE ALLOWED)

STUDENT NAME	
ID NUMBER	

Problem	Out of	Grade
1	**10**	
2	**8**	
3	**8**	
4	**8**	
5	**16**	
TOTAL	**50**	

1. (10 points) Answer the following:

   (a) Let $x = 7.477$ and $y = 3.789$ be 2 floating points in $\mathbb{F} = \mathbb{F}(10, 4, -2, +2)$. What is the relative Error in the computation of $x \oplus y$ in $\mathbb{F}$?

   (b) Let $x = 0.3721448693$ and $y = 0.3720214371$
   What is the relative error in the computation of $(fl_p(x) \ominus fl_p(y))$ in $\mathbb{F}$?

2. (8 points) Answer the following:

   (a) Let $x = [94F96A0]_{16}$. Is this a hexadecimal representation of some element in $\mathbb{F}_s$? Justify your answer.

   (b) In case $x \in \mathbb{F}_s$, determine the decimal number that has this hexadecimal representation in $\mathbb{F}_s$. Otherwise, modify x first, then find the required decimal number.

3. (8 points) Loss of significant figures may result in the computation of the following functions of the variable x for certain values of x. Specify these values then propose alternative functions that would remedy the loss of significant figures. (If necessary you may use Taylors series).

   (a) $f(x) = x - \sqrt{x^2 + 1}$

   (b) $g(x) = 1 - \cos(x/2)$

4. (8 points) Consider the polynomial $p(x) = 4 + x^2 + 3x^4 + 2x^8 - 5x^{16}$.

   (a) (4 points)) Put $p(x)$ in nested form.

   (b) (4 points) Find consequently the minimum number of floating-point operations to compute $p(x)$.

5. (16 points) Let $f(x) = \ln(1+x) - \frac{1}{x+1}$

   (a) (7 points) Prove that the function $f(x)$ has a unique positive root r in the interval $(0, 1)$:

   - (3 points) By plotting both functions $\ln(1+x)$ and $\frac{1}{x+1}$

   - (4 points) By studying the behaviour of $f(x)$ and $f'(x)$ on $[0, \infty)$

(b) (3 points) Apply 2 iterations of the Bisection method

- First Iteration: $r_1 =$

- Second Iteration: $r_2 =$

(c) (3 points) Write Newton's method iteration formula and apply 1 iteration of the formula using r_1 (found above) as initial choice.

(d) (3 points) Write the Secant method iteration formula and apply 1 iteration of the formula using r_1 and r_2 (found above) as initial choice.