AMERICAN UNIVERSITY OF BEIRUT
Faculty of Arts and Sciences
Mathematics Department

MATH 251
QUIZ II
FALL 2007-2008
Closed Book, 75 MINUTES

WRITE YOUR ANSWERS ON THE QUESTION SHEET

STUDENT NAME	
ID NUMBER	

Problem	Out of	Grade
1	16	
2	14	
3	12	
4	8	
TOTAL	50	

1. (16 points) Consider the following square matrix:

$$\begin{bmatrix} 2 & -2 & 3 & 7 \\ 4 & 4 & 0 & 7 \\ 2 & 1 & 1 & 3 \\ 6 & 5 & 4 & 17 \end{bmatrix}$$

(a) (9 points) Apply on this matrix, Gauss Elimination with **the scaled partial pivoting strategy**, showing the status of the 4 by 4 matrix after each elimination, i.e. each of the pivot rows and the corresponding multipliers should be identified and circled.

- Reduction 1:

Changing Matrix Coefficients Including Multipliers	scales s

(continued...)

(b) (7 points) Specify the elements of the 4 by 4 matrices P, L and U which verify:

$$PA = LU$$

and give the determinant of the original matrix A

2. (14 points) Consider the following $n * n$ **quadridiagonal matrix**

$$A = \begin{pmatrix} d_1 & u_1 & 0 & \cdots & \cdots & 0 & 0 & 0 \\ l_1 & d_2 & u_2 & 0 & \cdots & \cdots & 0 & 0 \\ v_1 & l_2 & d_3 & u_3 & 0 & \cdots & 0 & 0 \\ 0 & v_2 & l_3 & d_4 & u_4 & 0 & \cdots & 0 \\ 0 & 0 & v_3 & l_4 & d_5 & u_5 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & v_{n-3} & l_{n-2} & d_{n-1} & u_{n-1} \\ 0 & 0 & \cdots & 0 & 0 & v_{n-2} & l_{n-1} & d_n \end{pmatrix}$$

This matrix is assumed to have the diagonal dominance property:

$$|d_i| \geq |u_i| + |l_{i-1}| + |v_{i-2}|, \forall i, 3 \leq i \leq n-1$$

$$|d_1| \geq |u_1|, |d_2| \geq |u_2| + |l_1|, |d_n| \geq |l_{n-1}| + |v_{n-2}|$$

As such we can use naive Gauss elimination to reduce it into an upper triangular matrix.

(a) (8 points) Complete the following algorithm that should perform the reduction of the matrix A:

```
function [d,u,l,v]=lu_QuadriDiag(d,u,l,v)
% d is the diagonal vector. Its dimension is n
% u is the upper diagonal vector. Its dimension is n-1
% l is the first lower diagonal vector. Its dimension is n-1
% v is the second lower diagonal vector. Its dimension is n-2
for k= :
% Get the multipliers (take into consideration the last iteration)




% Modify the coefficients (take into consideration the last iteration)




end
```

(b) (4 points) Give the number of flops needed to execute the above algorithm.

(c) (2 points) Give the upper triangular matrix U obtained from this reduction.

3. (12 points) Consider a function $f(x)$ given at 6 distinct data points by the following table:

x	0	0.1	0.2	0.3	0.4	0.5	0.6
$f(x)$	1.00	1.01	1.02	1.05	1.08	1.13	1.19

(a) (8 points) In case you want to use this table to find an approximate value to $f(x)$, $0.2 < x < 0.3$, give on the basis of Newton's formula, the expression of $p_3(x)$, the most suitable interpolation polynomial of degree 3.

(b) (4 points) Compute $p_3(0.25)$.

4. (8 points) Consider the set:

$$D_3 = \{(x_i, y_i) | i = 1, 2, 3\},$$

where $y_i = f(x_i)$ for a given function f, ($x_i \neq x_j$, $i \neq j$).

x_1	x_2	x_3
y_1	y_2	y_3

Prove that:

$$[x_1, x_2, x_3] = [x_{i_1}, x_{i_2}, x_{i_3}]$$

where $\{i_1, i_2, i_3\}$ is any permutation of $\{1, 2, 3\}$.