STATICS: CE201

Chapter 6

Structural Analysis

Notes are prepared based on: Engineering Mechanics, Statics by R. C. Hibbeler, 12E Pearson

Dr M. Touahmia & Dr M. Boukendakdji
Civil Engineering Department, University of Hail
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6. Structural Analysis

Main Goal:

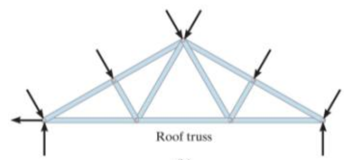
To show how to determine the forces in the members of a truss using the method of joints and the method of section.

Contents:

6.1 Simple Trusses

- A truss is a structure composed of slender members joined together at their end points.
- The members are usually wooden struts or metal bars.
- Planar trusses lie in a single plane and are often used to support roofs and bridges.
- The roof load is transmitted to the truss at the joints by means of a series of *purlins*.

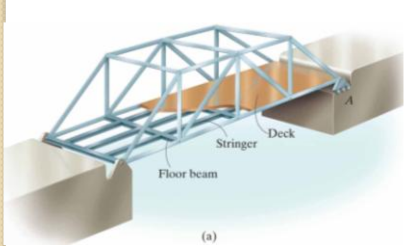
(a)



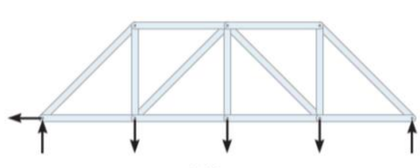
(b)

6.1 Simple Trusses

- The analysis of the forces developed in the truss members will be two-dimensional.
- In the case of a bridge, the load on the deck is first transmitted to stringers, then to floor beams, and finally to the joints of the two supporting side trusses.
- The bridge truss loading is also coplanar.



(a)



(b)

6.1 Simple Trusses

Assumption for Design:

- To design both the members and the connections of a truss, it is necessary first to determine the force developed in each member when the truss is subjected to a given loading.
- Each truss member is considered as a two-force member so that the forces at the ends of the member must be directed along the axis of the member.

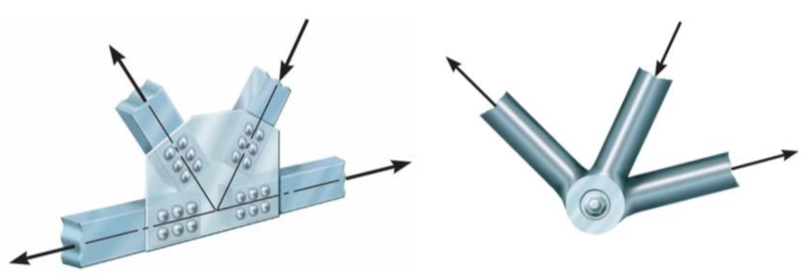
❑All loadings are applied at the joints.

❑The members are joined together by smooth pins.

6.1 Simple Trusses

Assumption for Design:

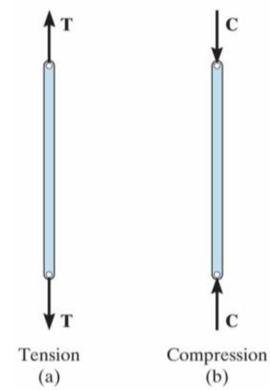
- The joint connections are usually formed by bolting or welding the ends of the members to a common plate, called a gusset plate, or by simply passing a large bolt or pin through each of the members.



6.1 Simple Trusses

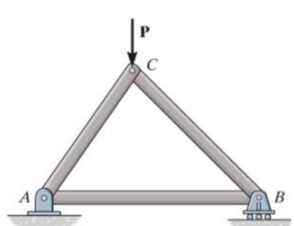
Assumption for Design:

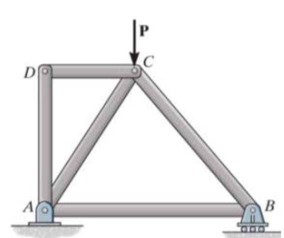
- The force acting at each end of the member is directed along the axis of the member.
- If this tends to elongate the member, it is a *tensile force* (T).
- If it tends to shorten the member, it is a *compressive force* (C).



6.1 Simple Trusses

- To prevent collapse, the form of a truss must be rigid. The simplest form which is rigid or stable is a triangle.
- Consequently, a simple truss is constructed by starting with a basic triangular element.
- Additional elements consisting of two members and a joint are added to the triangular element to form a simple truss.





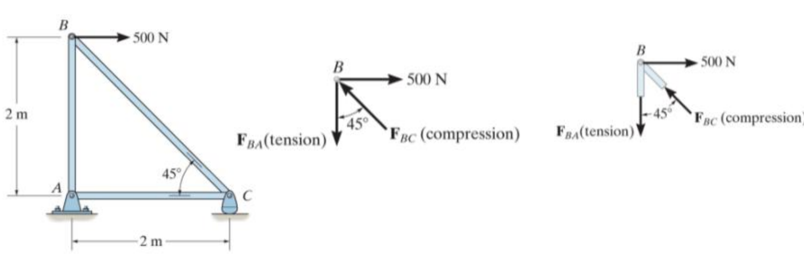
6.2 The Method of Joints

- The method of joint is based on the fact that if the entire truss is in equilibrium, then each of its joints is also in equilibrium.
- Therefore, if the free-body diagram of each joint is drawn, the force equilibrium equations can then be used to obtain the member forces acting on each joint.
- Since the members of a plane truss are straight two-force members lying in a single plane, each joint is subjected to a force system that is *coplanar* and *concurrent*.
- As a result, $\sum F_x = 0$ and $\sum F_y = 0$ need to be satisfied for equilibrium.

6.2 The Method of Joints

For example consider the pin at joint B of the truss:

- Three forces act on the pin: the 500 N force and the forces exerted by members *BA* and *BC*.
- $\mathbf{F}_{BA}$ is pulling on the pin, which means that member *BA* is in tension.
- $\mathbf{F}_{BC}$ is pushing on the pin, and consequently member *BC* is in compression.

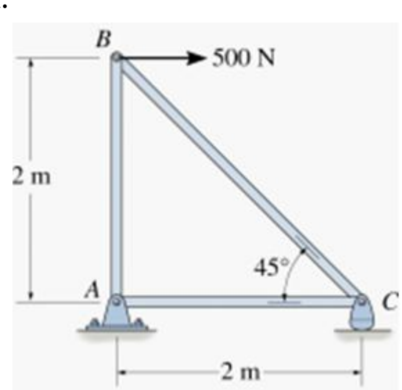


6.2 The Method of Joints

- When using the method of joints, always start at a joint having at least one known force and at most two unknown forces.
- $\sum F_x = 0$ and $\sum F_y = 0$ can be solved to determine the two unknowns.
- The correct sense of an unknown member force can be determined using one of two possible methods:
- The correct sense of direction of an unknown member force can be determined by inspection.
- Assume the unknown member forces acting on the joint's free-body diagram to be in tension.

Example 1

- Determine the force in each member of the truss and indicate whether the members are in tension or compression.

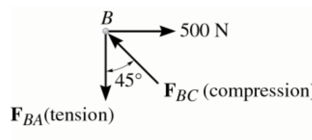


Solution 1

- Since we should have no more than two unknown forces at the joint and at least one known force acting there, we will begin our analysis at joint *B*.

Joint B:

- Applying the equations of equilibrium, we have



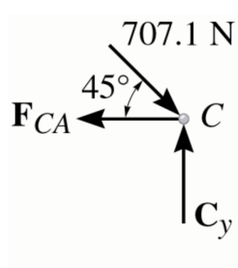
$$\sum F_x = 0 \qquad 500\text{N} - F_{BC}\sin 45^\circ = 0 \qquad F_{BC} = 707.1\text{N} \ \ (\text{C})$$

$$\sum F_y = 0 \qquad F_{BC}\cos 45^o - F_{BA} = 0 \qquad F_{BA} = 500\text{N} \quad (\text{T})$$

- Since the force in member *BC* has been calculated, we can proceed to analyze joint *C*

Solution 1

Joint C:

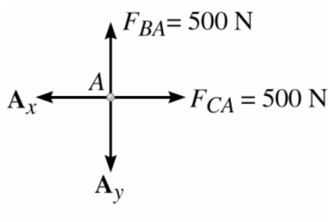


$$\sum F_x = 0 \qquad -F_{CA} + 707.1\cos 45^\circ\,\text{N} = 0 \qquad F_{CA} = 500\text{N} \ \ (\text{T})$$

$$\sum F_y = 0 \qquad C_y - 707.1\sin 45^\circ\,\text{N} = 0 \qquad C_y = 500\text{N}$$

Solution 1

Joint A:

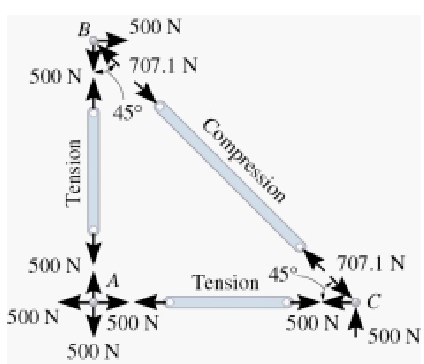


$\sum F_x = 0 \qquad 500N - A_x = 0 \qquad A_x = 500N$

$\sum F_y = 0 \qquad 500N - A_y = 0 \qquad A_y = 500N$

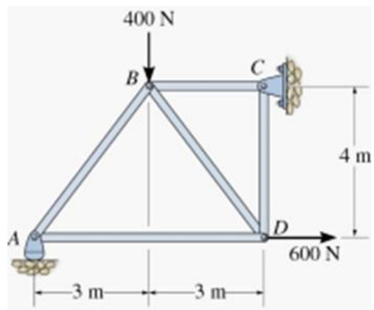
Solution 1

Note: The results of the analysis are summarized below, which shows the free-body diagram for each joint and member:



Example 2

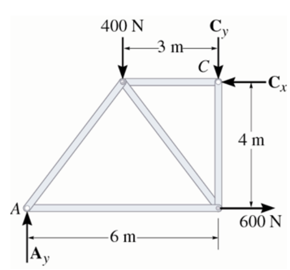
- Determine the force in each member of the truss and indicate whether the members are in tension or compression.



Solution 2

Support Reactions: No joint can be analyzed until the support reactions are determined, because each joint has more than three unknown forces acting on it.

Free-body diagram of the truss:



$$\sum F_x = 0$$

$$600N - C_x = 0 \qquad C_x = 600N$$

$$\sum M_C = 0$$

$$-A_y(6\,\text{m}) + 400\text{N}(3\,\text{m}) + 600\text{N}(4\,\text{m}) = 0 \qquad A_y = 600\text{ N}$$

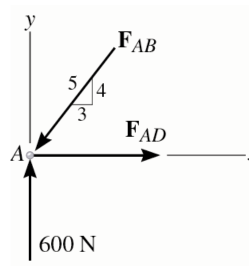
$$\sum F_y = 0 \qquad 600\text{N} - 400\text{N} - C_y = 0 \qquad C_y = 200\text{N}$$

Solution 2

- The analysis can now start at either joint *A* or *C*:

Joint A: $\mathbf{F}_{AB}$ is assumed to be compressive and $\mathbf{F}_{AD}$ is tensile

- Applying the equations of equilibrium, we have:

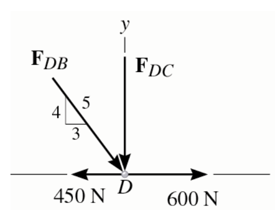


$$\sum F_y = 0 \qquad 600\text{N} - \frac{4}{5}F_{AB} = 0 \qquad F_{AB} = 750\text{ N}(C)$$

$$\sum F_x = 0 \qquad F_{AD} - \frac{3}{5}(750) = 0 \qquad F_{AD} = 450\text{ N}(\text{T})$$

Solution 2

Joint D: Use the result for $\mathbf{F}_{AD}$



$$\sum F_x = 0 \qquad -450\text{N} + \frac{3}{5}F_{DB} + 600\text{N} = 0$$

$$F_{DB} = -250\text{N}$$

- The negative sign indicates that $\mathbf{F}_{DB}$ acts in the opposite sense to that shown in the figure.

$$F_{DB} = 250\text{N (T)}$$

- To determine $\mathbf{F}_{DC}$, we can correct the sense of $\mathbf{F}_{DB}$ on the free body diagram.

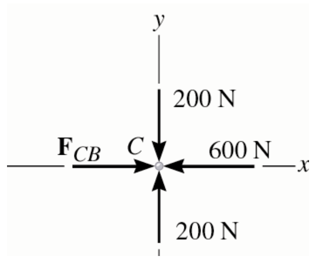
$$\sum F_y = 0 \qquad -F_{DC} - \frac{4}{5}(-250\text{N}) = 0 \qquad F_{DC} = 200\text{N (C)}$$

Solution 2

Joint C:

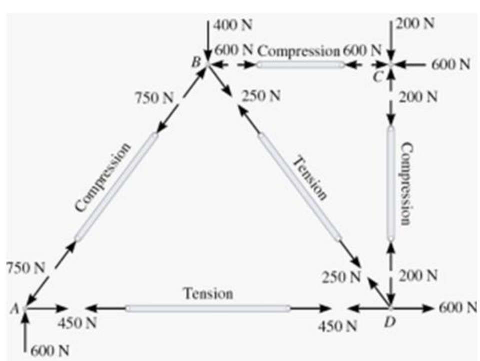
$$\sum F_x = 0 \qquad F_{CB} - 600\text{N} = 0 \qquad F_{CB} = 600\text{N (C)}$$

$$\sum F_y = 0 \qquad 200\text{N} - 200\text{N} = 0 \qquad \text{(Check)}$$



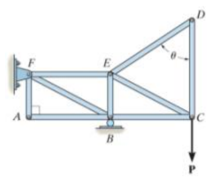
Solution 2

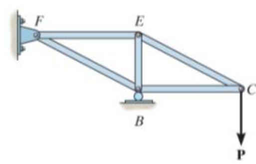
Note: The analysis is summarized below, which shows the free-body diagram for each joint and member:

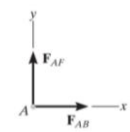


6.3 Zero-Force Members

- The *zero-force* members of a truss can generally be found by inspection of each of the joints.
- For example, consider the truss below.
- At **Joint *A***, it is seen that members *AB* and *AF* are zero-force members:







$\sum F_x = 0 \qquad F_{AB} = 0$

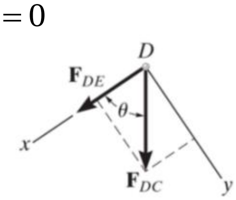
$\sum F_y = 0 \qquad F_{AF} = 0$

6.3 Zero-Force Members

- At **Joint *D***, it is seen that members *DC* and *DE* are zero-force members.

$\sum F_y = 0 \qquad F_{DC} \sin\theta = 0 \qquad F_{DC} = 0$

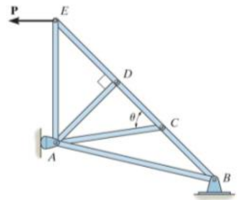
$\sum F_x = 0 \qquad F_{DE} = 0$

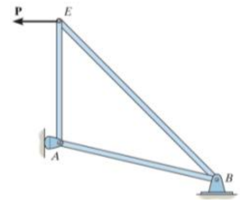


- From these observations, we can conclude that if only two members form a truss joint and no external load or support reaction is applied to the joint, the two members must be zero-force members.

6.3 Zero-Force Members

- Now consider this truss:

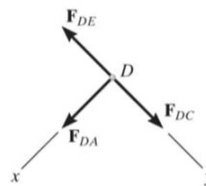




- At ***Joint D***, it is seen that *DA* is *zero-force* member:

$$\sum F_x = 0 \qquad F_{DA} = 0$$

$$\sum F_y = 0 \qquad F_{DC} = F_{DE}$$

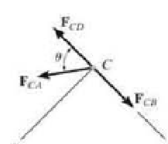


6.3 Zero-Force Members

- ***At Joint D***, it is seen that *DA* is zero-force member.

$$\sum F_x = 0 \qquad F_{CA} \sin\theta = 0 \qquad F_{CA} = 0$$

$$\sum F_y = 0 \qquad F_{CB} = F_{CD}$$



- In general then, if three members form a truss joint for which two of the members are collinear, the third member is a *zero-force member* provided no external force or support reaction is applied to the joint.

6.4 The Method of Sections

- When we need to find the force in only a few members of a truss we can analyze the truss using the *method of sections.*
- This method is based on the principle that if the truss is in equilibrium then any segment of the truss is also in equilibrium.
- If the forces within the members are to be determined, then an imaginary section can be used to cut each member into two parts.

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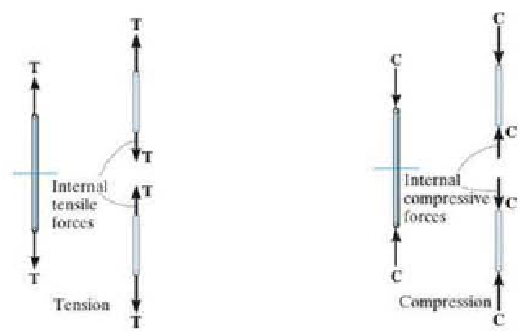
6.4 The Method of Sections



- The method of sections can also be used to cut or section the members of an entire truss.

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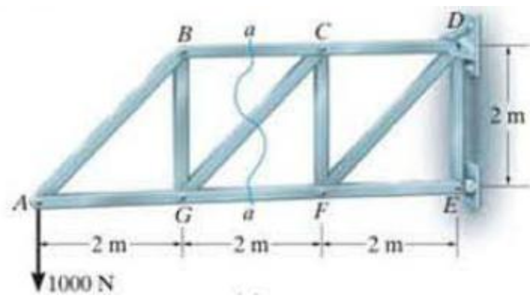
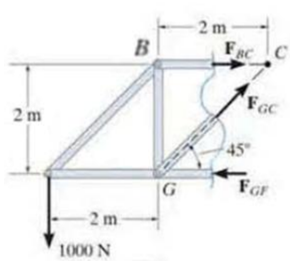
6.4 The Method of Sections

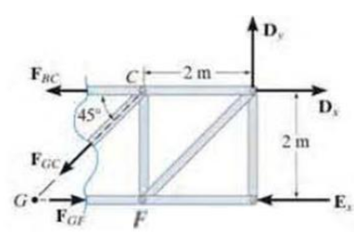
- For example, consider the truss below:
- If the forces in members *BC*, *GC*, and *GF* are to be determined, then section *a-a* would be appropriate.



6.4 The Method of Sections

- The free-body diagrams of the two segments are then:

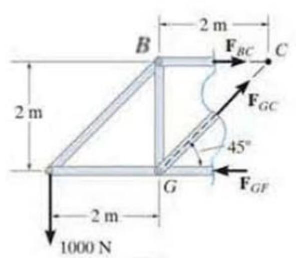


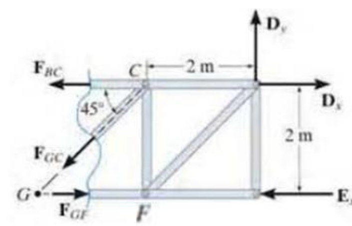


- Members *BC* and *GC* are assumed to be in tension since they are subjected to a pull, whereas *GF* in compression since it is subjected to a push.

6.4 The Method of Sections

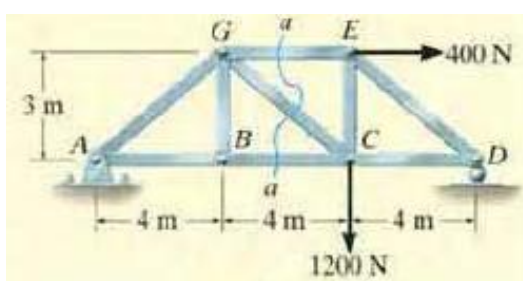
- The Three unknown member forces $\mathbf{F}_{BC}$, $\mathbf{F}_{GC}$, and $\mathbf{F}_{GF}$ can be obtained by applying the three equilibrium equations of the segments.
- The three support reactions $\mathbf{D_x}$, $\mathbf{D_y}$ and $\mathbf{E_x}$ can be obtained by considering a free-body diagram of the entire truss.





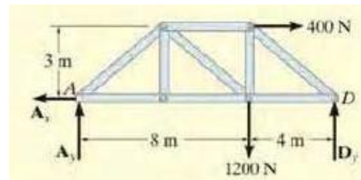
Example 2

- Determine the force in members GE, GC, and BC of the truss. Indicate whether the members are in tension or compression.



Solution 2

- Section *a-a* has been chosen since it cuts through the three members whose forces are to be determined.
- It is first necessary to determine the external reactions at *A* and *D*.



$\sum F_x = 0 \qquad 400\text{N} - A_x = 0$

$A_x = 400\text{N}$

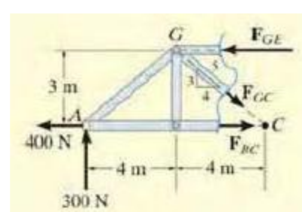
$\sum M_A = 0 \qquad -1200\text{N}(8\text{m}) - 400\text{N}(3\text{m}) + D_y(12\text{m}) = 0$

$D_y = 900\text{N}$

$\sum F_y = 0 \qquad A_y - 1200\text{N} + 900\text{N} = 0 \qquad A_y = 300\text{N}$

Solution 2

Free-Body Diagram: For the analysis the free-body diagram of the left portion of the sectioned truss will be used.



Equations of Equilibrium:

$\sum M_G = 0 \qquad -300\text{N}(4\text{m}) - 400\text{N}(3\text{m}) + F_{BC}(3\text{m}) = 0$

$F_{BC} = 800\text{N} \quad \text{(T)}$

$\sum M_C = 0 \qquad -300\text{N}(8\text{m}) + F_{GE}(3\text{m}) = 0 \qquad F_{GE} = 800\text{N} \quad \text{(C)}$

$\sum F_y = 0 \qquad 300\text{N} - \frac{3}{5}F_{GC} = 0 \qquad F_{GC} = 500\text{N} \quad \text{(T)}$