QUIZ 1 – SOLUTION
SPRING 2013-14
(Monday March 17, 2014)
CIVE210 – STATICS
CLOSED BOOK, 1 HR 30 MN

Name: # **ID#:** 201300000 **Sec:** 1, 2, 3, 24

NOTES

- 3 PROBLEMS– 11 PAGES.
- ALL YOUR ANSWERS SHOULD BE PROVIDED ON THE QUESTION SHEETS.
- **TWO EXTRA SHEETS ARE PROVIDED AT THE END.**
- **ASK FOR ADDITIONAL SHEETS IF YOU NEED MORE SPACE.**
- SOME ANSWERS MAY REQUIRE MUCH LESS THAN THE SPACE PROVIDED.
- ***DO NOT*** USE THE BACK OF THE SHEETS FOR ANSWERS.
- *DRAFT BOOKLET WILL BE PROVIDED; BUT DO NOT USE FOR ANSWERS.*
- BOTH QUESTION SHEETS AND DRAFT BOOKLET SHOULD BE RETURNED.
- CHECK BOXES ARE TO CONFIRM THAT YOU HAVE SOLVED A QUESTION. ☑

YOUR COMMENT(S)

DO NOT WRITE IN THE SPACE BELOW

MY COMMENT(S)

YOUR GRADE

Problem I:	___ /25
Problem II:	___ /35
Problem III:	___ /40
Bonus/Extras – Organization, Neatness, Special, ...:	___
TOTAL:	/100

Problem I: *(25 points)*

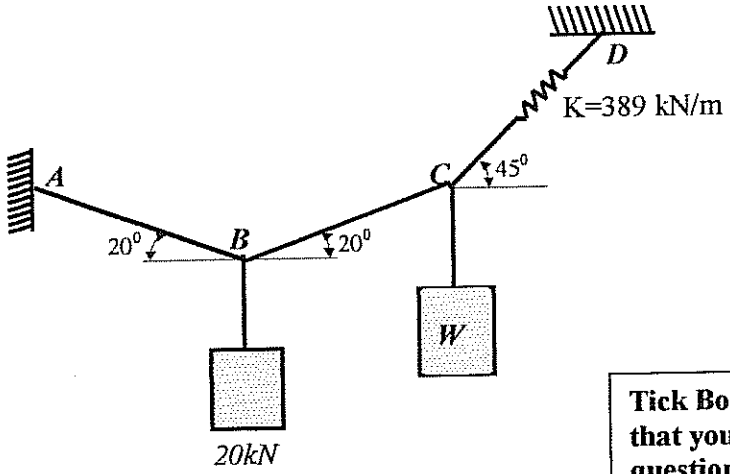
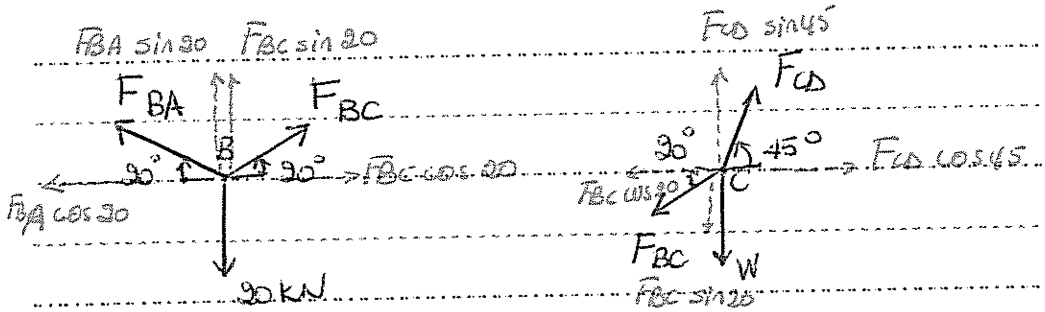


Figure I

Tick Boxes to check that you solved all questions

Referring to Figure I, the system with weights, cables, and spring is in its equilibrium position. Determine the forces in the cables and spring, the weight W, and the stretch (or extension) of the spring. *(25 points)* ☐

Calculations and/or Diagrams:



F.B.D at B F.B.D at C

Equilibrium at B:

$\xrightarrow{+} \sum F_x = 0 \Rightarrow -F_{BA}\cos 20 + F_{BC}\cos 20 = 0 \Rightarrow \boxed{F_{BA} = F_{BC}}$ ... Eq.(1)

$+\uparrow \sum F_y = 0 \Rightarrow F_{BA}\sin 20 + F_{BC}\sin 20 - 20 = 0$ ...... Eq.(2)

substitute Eq.(1) in Eq.(2)

$\Rightarrow F_{BC}\sin 20 + F_{BC}\sin 20 = 20$

$\Rightarrow F_{BC} = \dfrac{20}{2(\sin 20)} = 29.24$ kN & $\boxed{F_{BA} = 29.24}$

$\boxed{F_{BC} = 29.24}$

Calculations and/or Diagrams (cont'd):

Equilibrium at C:

$\xrightarrow{+} \Sigma F_x = 0 \Rightarrow -F_{BC} \cos 20 + F_{CD} \cos 45 = 0$

$\Rightarrow -29.24 \cos 20 + F_{CD} \cos 45 = 0$

$\Rightarrow \boxed{F_{CD} = 38.86 \text{ KN}}$

the stretch of the spring : $F_{CD} = K.s$

$38.86 = 389 . s$

$\Rightarrow \boxed{s \cong 0.1 \text{ m}}$

$+\uparrow \Sigma F_y = 0 \Rightarrow -F_{BC} \sin 20 + F_{CD} \sin 45 - W = 0$

$-29.24 \sin 20 + 38.86 \sin 45 = W$

$\Rightarrow \boxed{W = 17.48 \text{ KN}}$

Problem II: *(35 points)*

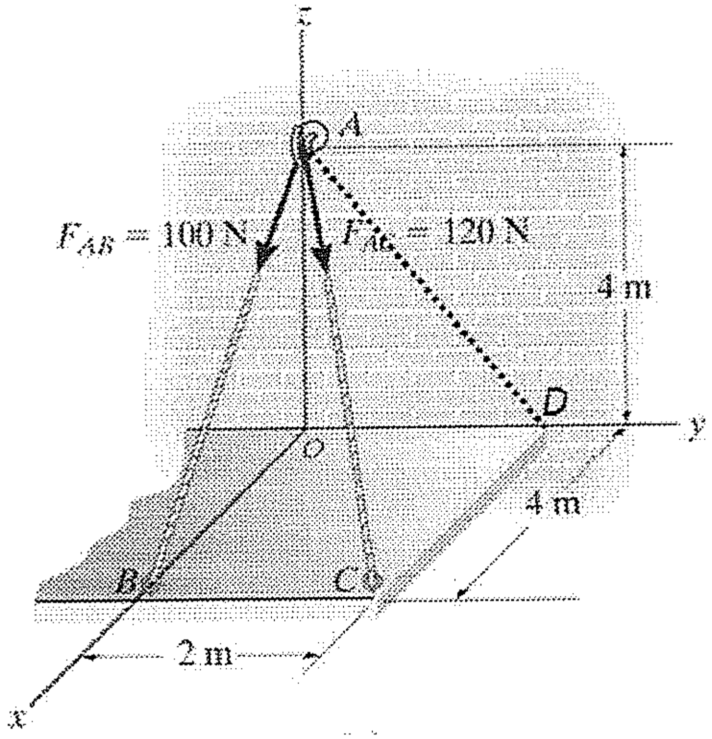


Figure II

Referring to Figure II:

1. Compute the resultant force of the two cable forces at point A and express in Cartesian vector form; then compute its magnitude and direction. *(15 points)*.
2. Compute the component of this resultant force on line AD, and express it as a Cartesian vector. *(10 points)*
3. Determine the component of the resultant moment from the two cable forces along line OD. Express the moment in Cartesian form and sketch it. *(10 points)*

Calculations and/or Diagrams:

1– Express F_{AC} and F_{AB} in Cartesian vector.

Coordinates A(0,0,4) , B (4,0,0) & C (4,2,0)

$\vec{F}_{AB} = F_{AB}\, u_{AB}$

$\vec{u}_{AB} = \dfrac{4\vec{i} + 0\vec{j} - 4\vec{k}}{\sqrt{(4)^2 + (0)^2 + (4)^2}} = 0.707\,\vec{i} + 0\vec{j} - 0.707\,\vec{k}$

$\therefore \vec{F}_{AB} = 100\,(0.707\,\vec{i} + 0\vec{j} - 0.707\,\vec{k}) = 70.7\,\vec{i} + 0\vec{j} - 70.7\,\vec{k}$

$\boxed{\vec{F}_{AB} = 70.7\,\vec{i} + 0\vec{j} - 70.7\,\vec{k}}$

Calculations and/or Diagrams (cont'd):

$\vec{F}_{AC} = F_{AC}\,\vec{u}_{AC}$

$\vec{u}_{AC} = \dfrac{4\vec{i} + 2\vec{j} - 4\vec{k}}{\sqrt{(4)^2 + (2)^2 + (-4)^2}} = \dfrac{2}{3}\vec{i} + \dfrac{1}{3}\vec{j} - \dfrac{2}{3}\vec{k}$

$\vec{F}_{AC} = 120\left(\dfrac{2}{3}\vec{i} + \dfrac{1}{3}\vec{j} - \dfrac{2}{3}\vec{k}\right) \quad \therefore \boxed{\vec{F}_{AC} = \{80\vec{i} + 40\vec{j} - 80\vec{k}\}\ N}$

Resultant Force:

$F_{Rx} = F_{AB_x} + F_{AC_x} = 70.7 + 80 = 150.7\ N$

$F_{Ry} = F_{AB_y} + F_{AC_y} = 0 + 40 = 40\ N$

$F_{Rz} = F_{AB_z} + F_{AC_z} = -70.7 - 80 = -150.7\ N$

Magnitude: $F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2 + F_{Rz}^2} = \sqrt{(150.7)^2 + (40)^2 + (-150.7)^2} = 216.84\ N$

$\therefore \boxed{F_R = 216.84\ N}$

$\boxed{\vec{F}_R = \{150.7\vec{i} + 40\vec{j} - 150.7\vec{k}\}\ N}$

Direction:

$\cos\alpha = \dfrac{F_{Rx}}{F_R} = \dfrac{150.7}{216.84} \Rightarrow \boxed{\alpha = 45.98^\circ}$

$\cos\beta = \dfrac{F_{Ry}}{F_R} = \dfrac{40}{216.84} \Rightarrow \boxed{\beta = 79.37^\circ}$

$\cos\gamma = \dfrac{F_{Rz}}{F_R} = \dfrac{-150.7}{216.84} \Rightarrow \boxed{\gamma = 134.03^\circ}$

2. $F_{R/AD} = \vec{F}_R \cdot \vec{u}_{AD}$; coordinates $D(0, 2, 0)$

$\vec{u}_{AD} = \dfrac{0\vec{i} + 2\vec{j} - 4\vec{k}}{\sqrt{(0)^2 + (2)^2 + (-4)^2}} = 0\vec{i} + 0.447\vec{j} - 0.894\vec{k}$

$\therefore F_{R/AD} = \{150.7\vec{i} + 40\vec{j} - 150.7\vec{k}\} \cdot \{0\vec{i} + 0.447\vec{j} - 0.894\vec{k}\} = 152.61$

$\therefore \boxed{F_{R/AD} = 152.61\ N}$

Calculations and/or Diagrams (cont'd):

In Cartesian Vector:

$$\vec{F}_{R/AD} = F_{R/AD}\,\vec{u}_{AD} = 152.61\left\{0\vec{i} + 0.447\vec{j} - 0.894\vec{k}\right\}$$

$$\therefore \boxed{\vec{F}_{R/AD} = \left\{0\vec{i} + 68.22\vec{j} - 136.44\vec{k}\right\}^{N}}$$

3.

$$M_{R/OD} = \vec{u}_{OD}\cdot\left(\vec{r}_{OA} \times \vec{F}_{AC}\right) + \vec{u}_{OD}\cdot\left(\vec{r}_{OA} \times \vec{F}_{AB}\right) = \vec{u}_{OD}\cdot\left(\vec{r}_{OA} \times \vec{F}_{R}\right)$$

* the position vector $\vec{r}$ could be either $\vec{r}_{OA}$, $\vec{r}_{OB}$, $\vec{r}_{DA}$ or $\vec{r}_{DB}$ for $\vec{F}_{AB}$ and either $\vec{r}_{OA}$, $\vec{r}_{OC}$, $\vec{r}_{DA}$ or $\vec{r}_{DC}$ for $\vec{F}_{AC}$

thus, $\vec{r}_{OA}$ is selected for both forces

where:

$$\vec{r}_{OA} = \vec{r}_{A} = \left\{0\vec{i} + 0\vec{j} + 4\vec{k}\right\}^{m}$$

$$\vec{u}_{OD} = 0\vec{i} + 1\vec{j} + 0\vec{k} = \vec{u}_{y}$$

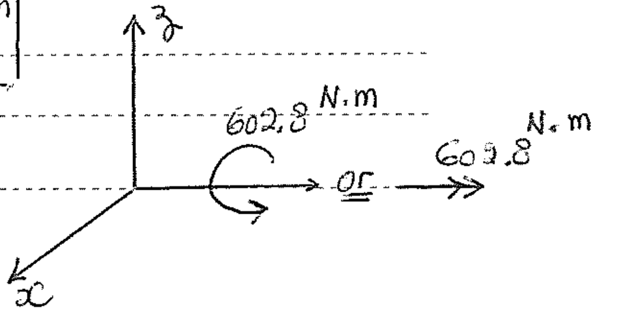
$$M_{R/OD} = \vec{u}_{OD}\cdot\left(\vec{r}_{OA} \times \vec{F}_{R}\right)$$

$$M_{R/OD} = \begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 4 \\ 150.7 & 40 & -150.7 \end{vmatrix} = +4(150.7) \Rightarrow \boxed{M_{R/OD} = 602.8\ N.m}$$

In Cartesian vector: $\vec{M}_{R/OD} = M_{R/OD}\,\vec{u}_{OD} = 602.8\left\{0\vec{i} + 1\vec{j} + 0\vec{k}\right\}$

$$\boxed{\vec{M}_{R/OD} = \left\{0\vec{i} + 602.8\vec{j} + 0\vec{k}\right\}^{N.m}}$$

$$M_{R/OD} = M_y = 602.8\ N.m$$



Problem III: *(40 points)*

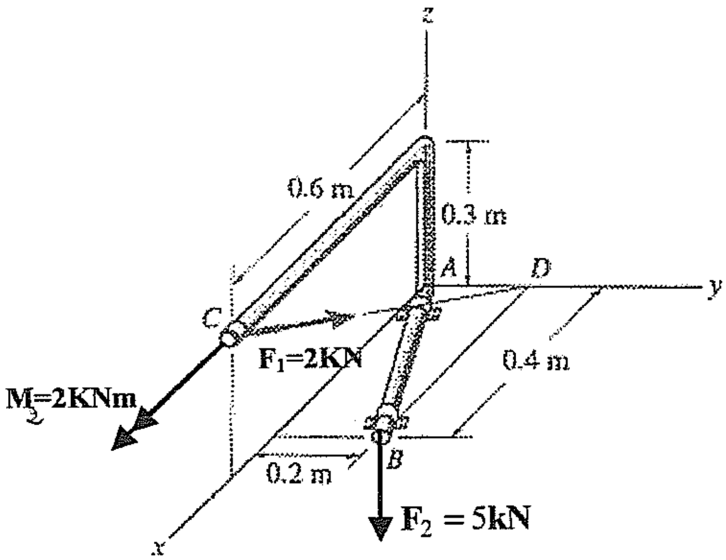


Figure III

The rigid pipe system is subjected to the forces and moment as shown in Figure III. Note that F_2 is in the vertical direction and M_2 is about the X axis.

1. Compute the resultant moment at point A and express in Cartesian vector form; then compute its magnitude and direction. *(15 points).* ☐
2. Determine the component of this moment about an axis extending between points A and B. Express the result as Cartesian vector. *(15 points)* ☐
3. Using a scalar approach, compute the moment at point A in each of the X, Y, and Z direction and compare with Question 1. *(10 points)* ☐

Calculations and/or Diagrams:

1- $\vec{M_A} = \vec{M_2} + (\underbrace{\vec{r_{AC}} \times \vec{F_1}}_{\text{or } \vec{r_{AD}} \times \vec{F_1}}) + (\vec{r_{AB}} \times \vec{F_2})$

$\vec{M_2} = \{2\vec{i} + 0\vec{j} + 0\vec{k}\}$ KN.m

coordinates: A(0, 0, 0) B(0.4, 0.2, 0) C(0.6, 0, 0.3). D(0, 0.2, 0

position vectors:

$\vec{r_{AC}} = 0.6\vec{i} + 0\vec{j} + 0.3\vec{k}$

$\vec{r_{AB}} = 0.4\vec{i} + 0.2\vec{j} + 0\vec{k}$

Express F_1 and F_2 in Cartesian vector:

$\vec{F_1} = F_1 \vec{u_{CD}} = 2\left\{-\frac{6}{7}\vec{i} + \frac{2}{7}\vec{j} - \frac{3}{7}\vec{k}\right\} \Rightarrow \vec{F_1} = \left\{-\frac{12}{7}\vec{i} + \frac{4}{7}\vec{j} - \frac{6}{7}\vec{k}\right.$

where:

$$\vec{u_{CD}} = \frac{-0.6\vec{i} + 0.2\vec{j} - 0.3\vec{k}}{\sqrt{(-0.6)^2 + (0.2)^2 + (-0.3)^2}} = -\frac{6}{7}\vec{i} + \frac{2}{7}\vec{j} - \frac{3}{7}\vec{k}$$

Calculations and/or Diagrams (cont'd):

$\vec{F_2} = F_2(0\vec{i} + 0\vec{j} - 1\vec{k}\} = 0\vec{i} + 0\vec{j} - 5\vec{k}$ $\therefore$ $\vec{F_2} = \{0\vec{i} + 0\vec{j} - 5\vec{k}\}$ KN

$$\vec{M_A} = \{2\vec{i} + 0\vec{j} + 0\vec{k}\} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0.6 & 0 & 0.3 \\ \frac{-12}{7} & \frac{4}{7} & \frac{-6}{7} \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0.4 & 0.2 & 0 \\ 0 & 0 & -5 \end{vmatrix}$$

$$= \{2\vec{i} + 0\vec{j} + 0\vec{k}\} + \left\{\frac{-1.2}{7}\vec{i} + 0\ \vec{j} + \frac{2.4}{7}\vec{k}\right\} + \{-\vec{i} + 2\vec{j} + 0\vec{k}\}$$

$$\boxed{\vec{M_A} = \{0.829\vec{i} + 2.0\vec{j} + 0.343\vec{k}\} \text{ KN.m}}$$

Magnitude and direction:

$M_A = \sqrt{(0.829)^2 + (2.0)^2 + (0.343)^2} = 2.192$ KN.m

$\cos\alpha = \frac{0.829}{2.192} \Rightarrow \boxed{\alpha = 67.8°}$

$\cos\beta = \frac{2.0}{2.192} \Rightarrow \boxed{\beta = 24.16°}$

$\cos\gamma = \frac{0.343}{2.192} \Rightarrow \boxed{\gamma = 81.0°}$

2- $M_{A/AB} = \vec{M_A} \cdot \vec{u_{AB}} = \{0.829\vec{i} + 2.0\vec{j} + 0.343\}\{0.895\vec{i} + 0.447\vec{j} + 0\vec{k}\}$

where: $\Rightarrow \boxed{M_{A/AB} = 1.636 \text{ KN.m}}$

$\vec{u_{AB}} = \frac{0.4\vec{i} + 0.2\vec{j} + 0\vec{k}}{\sqrt{(0.4)^2 + (0.2)^2 + (0)^2}} = 0.895\vec{i} + 0.447\vec{j} + 0\vec{k}$

In Cartesian Vector:

$\vec{M_{A/AB}} = M_{A/AB}\ \vec{u}_{AB} = 1.636\ \{0.895\vec{i} + 0.447\vec{j} + 0\vec{k}\}$

$$\Rightarrow \boxed{\vec{M_{A/AB}} = \{1.464\vec{i} + 0.732\vec{j} + 0\vec{k}\} \text{ KN.m}}$$

Calculations and/or Diagrams (cont'd):

3. Scalar approach:

$M_x = -5(0.2) - \frac{4}{7}(0.3) + 2 = 0.829$ KN.m

$M_y = 5(0.4) - \frac{12}{7}(0.3) + \frac{6}{7}(0.6) = 2$ KN.m

$M_z = \frac{4}{7}(0.6) = 0.343$ KN.m

Results are the same as Question (1).