QUIZ 2

Fall 2013-14

(Wednesday November 27, 2013)

CIVE210 – STATICS

CLOSED BOOK, 1 HR 25 MN

Name: **** **** (☺) **ID#:** 201***** **Section:** 1,2,3,4 & 5

NOTES

- 2 PROBLEMS– 10 PAGES.
- ALL YOUR ANSWERS SHOULD BE PROVIDED ON THE QUESTION SHEETS.
- TWO EXTRA SHEETS IS PROVIDED AT THE END.
- ASK FOR ADDITIONAL SHEETS IF YOU NEED MORE SPACE.
- SOME ANSWERS MAY REQUIRE MUCH LESS THAN THE SPACE PROVIDED.
- ***DO NOT*** USE THE BACK OF THE SHEETS FOR ANSWERS.
- *DRAFT BOOKLET WILL BE PROVIDED; BUT DO NOT USE FOR ANSWERS.*
- BOTH QUESTION SHEETS AND DRAFT BOOKLET SHOULD BE RETURNED.
- CHECK BOXES ARE TO CONFIRM THAT YOU HAVE SOLVED A QUESTION. ☑

YOUR COMMENT(S)

DO NOT WRITE IN THE SPACE BELOW

MY COMMENT(S)

YOUR GRADE

Problem I: ___/35

Problem II: ___/65

Bonus/Extras – Organization, Neatness, Special, ...: ___

TOTAL: /100

Problem I: *(35 points)*

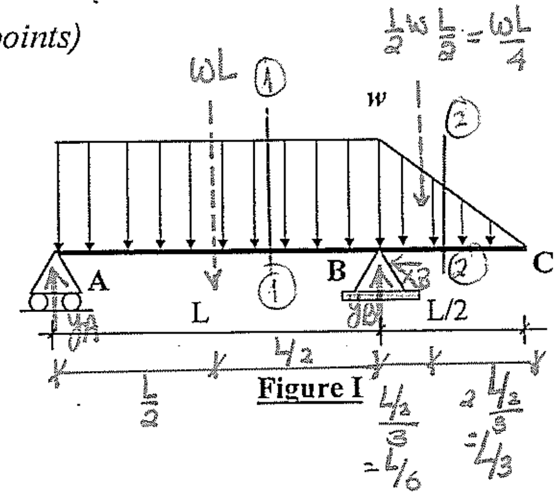
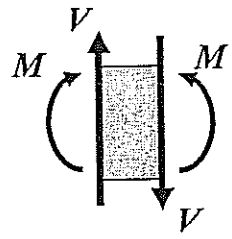


Figure I



(+) Convention

Tick Boxes to check that you solved all questions

For the beam shown in Figure I:

1- Compute the reactions at point A and B. *(7 points)*
2- Write the equations for shear force and bending moment in the beam, and draw the shear and moment diagrams. *(20 points)*
3- For L = 6 m, determine the largest intensity of w (kN/m) that the beam can support if the beam can withstand a maximum absolute (positive or negative) shear force of Vmax= 9 kN and a maximum absolute (positive or negative) moment of Mmax = 6 kN.m. *(8 points)*

Calculations and/or Diagrams:

1- Reactions

$+\circlearrowleft \sum M_A = 0 \Rightarrow -\omega L\left(\frac{L}{2}\right) + y_B(L) - \frac{\omega L}{4}\left(\frac{L}{6}+L\right) = 0$

$\Rightarrow y_B = \frac{1}{L}\left[\frac{\omega L^2}{2} + \frac{7\omega L^2}{24}\right] = \frac{19\omega L}{24}$

$\therefore \boxed{y_B = \frac{19\omega L}{24} \uparrow}$

$+\uparrow \sum F_y = 0 \Rightarrow y_A - \omega L + \frac{19\omega L}{24} - \frac{\omega L}{4} = 0$

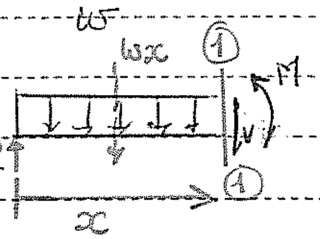
$\Rightarrow y_A = \frac{5\omega L}{4} - \frac{19\omega L}{24} = \frac{11}{24}\omega L$

$\therefore \boxed{y_A = \frac{11}{24}\omega L \uparrow}$

$\xrightarrow{+} \sum F_x = 0 \Rightarrow \boxed{x_B = 0}$

Calculations and/or Diagrams (cont'd):

2. Section ①① (Left) $0 < x < L$



* $V = \frac{11}{24}wL - wx$ (1st order) — -ve cst.

slope of shear diagram -ve const.

at $x = 0 \Rightarrow V = V_A = \frac{11}{24}wL$

at $x = L \Rightarrow V = V_{B_L} = \frac{11}{24}wL - wL = -\frac{13}{24}wL$

* $M = \frac{11}{24}wL(x) - wx\left(\frac{x}{2}\right) = \frac{11}{24}wLx - w\frac{x^2}{2}$ (2nd order)

at $x = 0$ $M = M_A = 0$

at $x = L$ $M = M_B = \frac{11}{24}wL^2 - \frac{wL^2}{2} = -\frac{wL^2}{24}$

$\frac{dM}{dX} = \frac{11}{24}wL - wx$ $\begin{cases} \text{at } x = 0 \Rightarrow \frac{dM}{dX} = \frac{11wL}{24} \\ \text{at } x = L \Rightarrow \frac{dM}{dX} = -\frac{13wL}{24} \end{cases}$

Concavity $\frac{d^2M}{dX^2} = -w \Rightarrow$ -ve (⌢) "always -ve"

* $M_{max} = ?$ value and Location

M_{max} at $V = 0$

$\Rightarrow V = \frac{11}{24}wL - wx = 0 \Rightarrow x = \frac{11L}{24}$

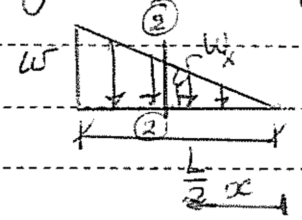
$\therefore M_{max} = \frac{11}{24}wL\left(\frac{11}{24}L\right) - \frac{w}{2}\left(\frac{11L}{24}\right)^2 = \frac{1}{2}\,\frac{121}{576}\,wL^2$

$\therefore M_{max} = 0.105\,wL^2$

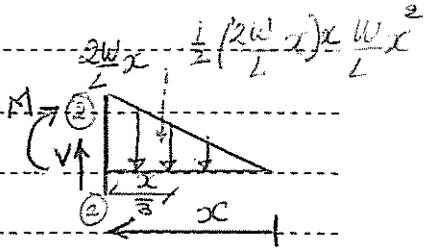
Calculations and/or Diagrams (cont'd):

Sec ②② (Right) $0 < x < \frac{L}{2}$

By similar triangle



$\frac{w}{L/2} = \frac{w_x}{x} \Rightarrow w_x = \frac{2w}{L}x$



* $V = (+)\frac{w}{L}x^2$ (2nd order)

slope of S.FD (+)

* $M = -\frac{wx^2}{L}\left(\frac{x}{3}\right) = \frac{-wx^3}{3L}$ (3rd order)

at $x = 0 \Rightarrow V = V_C = 0$
at $x = L/2 \Rightarrow V = V_{BR} = \frac{w}{L}\left(\frac{L}{2}\right)^2 = \frac{wL}{4}$

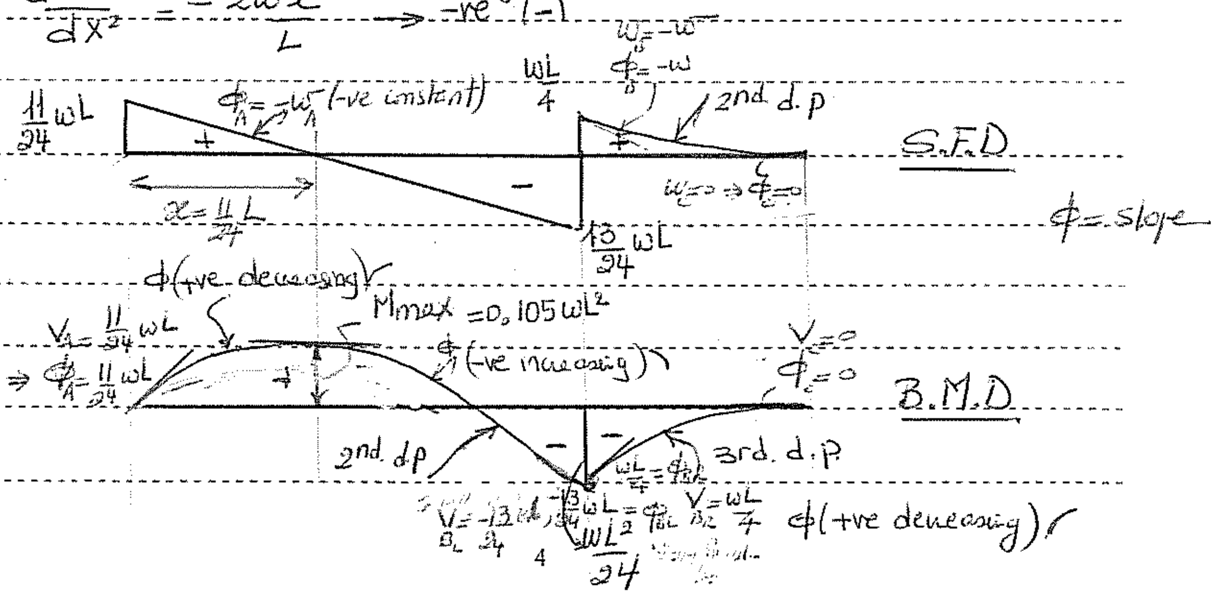
$\frac{dV}{dx} = \frac{+2wx}{L} \Rightarrow$ at $x=0$ $\frac{dV}{dx} = 0 \therefore \phi = 0$; at $x = \frac{L}{2}$ $\frac{dV}{dx} = w \therefore \phi = w$

at $x = 0 \Rightarrow M = M_C = 0$
at $x = \frac{L}{2} \Rightarrow M = M_B = -\frac{w}{3L}\left(\frac{L}{2}\right)^3 = -\frac{wL^2}{24}$ (match with M_B -sec-1 OK)

$\frac{dM}{dx} = -\frac{wx^2}{L}$

Concavity $\frac{d^2M}{dx^2} = -\frac{2wx}{L} \rightarrow$ allway -ve (⌢)

Diagrams.



Calculations and/or Diagrams (cont'd):

3_ for $L = 6$ maximum intensity $w = ?$

* $V_{max} = \frac{13}{24} wL = 9 \text{ KN} \Rightarrow \frac{13}{24} w(6) = 9 \text{ KN}$

$\Rightarrow w = 2.77 \text{ KN/m}$

* $M_{max} = 0.105 wL^2 = 6 \text{ KN.m} \Rightarrow 0.105 w(6)^2 = 6 \text{ KN.m}$

$\Rightarrow w = 1.58 \text{ KN/m}$

∴ the largest intensity the beam can support so that the shear and moment will not exceed the maximum allowable values is:

$$w_{max} = 1.58 \text{ KN/m}$$

Problem II: *(65 points)*

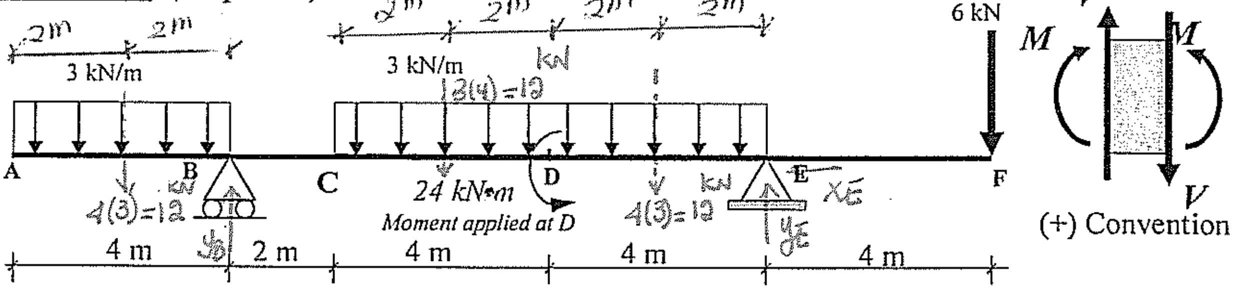


Figure II

Tick Boxes to check that you solved all questions

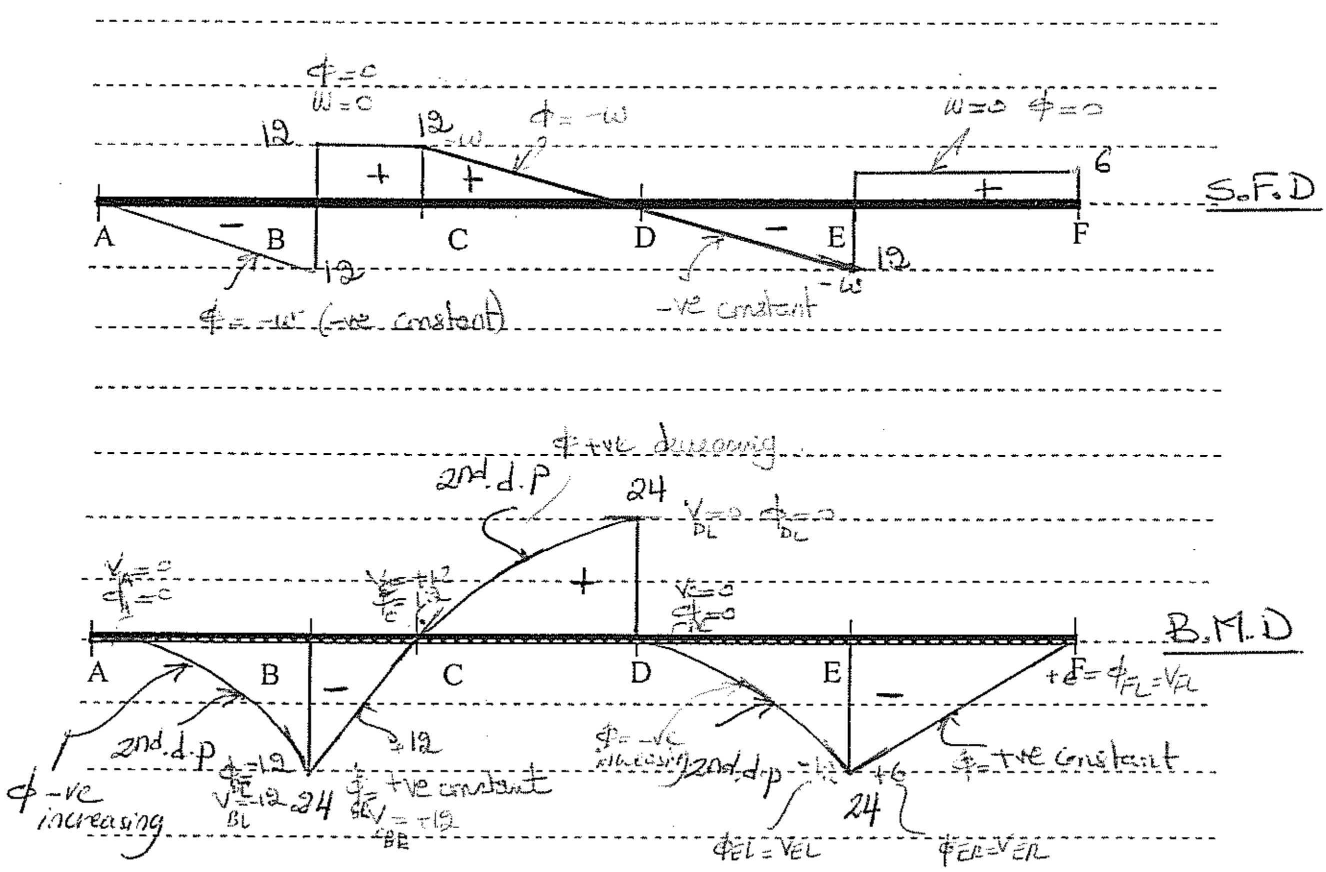
For the beam shown in Figure II:

1- Compute the reactions at points A (!), B, and E. *(CHECK THEM MORE THAN ONE TIME BEFORE YOU CONTINUE) (10 points)* ☑

2- Using sections, compute the shear force and bending moments at points A, B, C, D, E, and F. *(15 points)* ☑

3- Using the method of integration (or areas), draw the shear force and bending moment diagrams (use the space provided below for the diagrams and draw to scale as much as you can). Show the important and necessary features and values on the diagrams and indicate the maximum positive and negative shears and moments in the beam. *(40 points)* ☑

Calculations and/or Diagrams: (LEAVE THIS PAGE CLEAN FOR YOUR DIAGRAMS)



Calculations and/or Diagrams (cont'd):

1- Reactions:

$+\circlearrowleft \sum M_B = 0 \Rightarrow 12(12) - 12(4) + 24 - 12(8) + y_E(10) - 6(14) = 0$

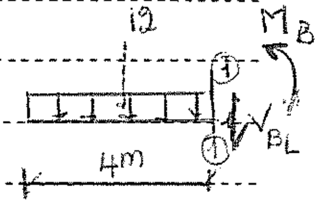
$\Rightarrow \boxed{y_E = 18\ kN \uparrow}$

$+\uparrow \sum F_y = 0 \Rightarrow 12 - 12 + y_A - 12 + 18 - 6 = 0 \Rightarrow \boxed{y_A = 24\ kN \uparrow}$

$\overset{+}{\rightarrow} \sum F_x = 0 \Rightarrow \boxed{x_E = 0}$

2- * Internal forces at A:

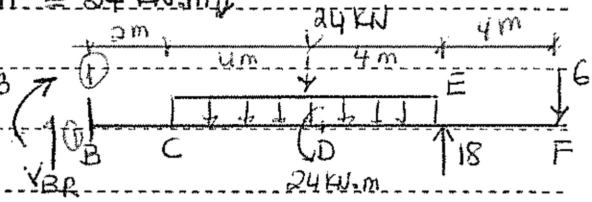
$V_A = 0 \quad M_A = 0$



Internal forces at B: sec ①-① (Left side)

$+\uparrow \sum F_y = 0 \quad V_B = V_{BL} = -12\ kN$

$+\circlearrowleft \sum M = 0 \quad M = -12(12) = -24\ kN.m = 24\ kN.m \circlearrowright$

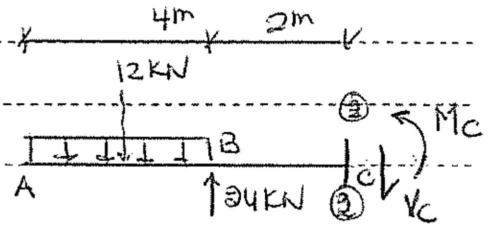


sec ①-① (Right side)

$V_B^+ = V_{BR} = 6 - 18 + 24 = 12\ kN$

$\Delta V_B = V_{BL} - V_{BR} = \text{Jump at } B\ (y_B = 24\ kN) \therefore ok.$

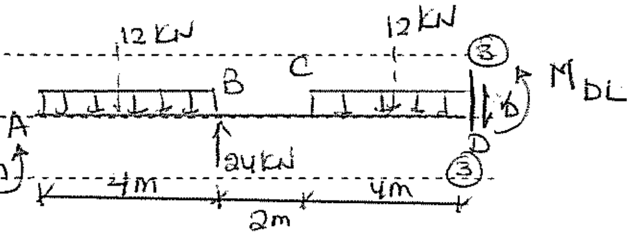
check! $M_B = -6(14) + 18(10) - 24(6) + 24 = -24\ kN.m \circlearrowright$ same as left side section ∴ ok.



* Internal forces at C sec ②-②

$\sum F_y = 0\ V_C = -12 + 24 = +12\ kN$

$\sum M = 0 \Rightarrow M_C = -12(4) + 24(2) = 0$



* Internal forces at D (left side)

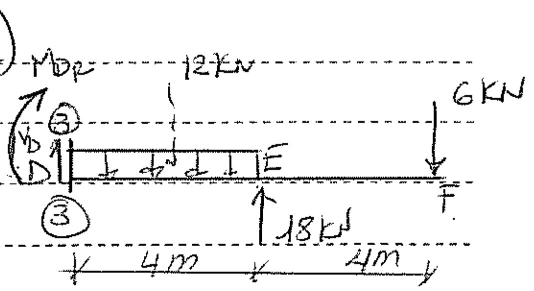
$V_D = -12 + 24 - 12 = 0\ kN$

$M_{DL} = -12(8) + 24(6) - 12(2) = 24\ kN.m \circlearrowleft$

Calculations and/or Diagrams (cont'd):

* Internal forces at D (Right side)

Sec (3)(3)



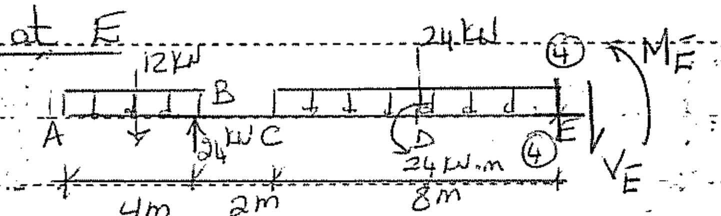
$V_D = +6 - 18 + 12 = 0$ (same as above ∴ ok)

$M_{DR} = -6(8) + 18(4) - 12(2) = 0$

$\Delta M_D = M_{DL} + M_{DR}$ = Jump at D "concentrated moment at D = 24 kN.m ∴ ok.

* Internal forces at E

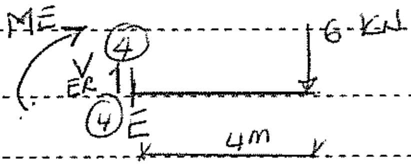
Left side section



$V_{E_L} = -12 + 24 - 24 = -12$ kN

$M_E = -12(12) + 24(10) - 24(4) - 24 = -24$ kN.m

Right side section



$V_{E_R} = +6$ kN

$M_E = -6(4) = -24$ kN.m

* Internal forces at F

$M_F = 0$ (free end)

ΔV_F { $V_{F_L} = 6$ kN (concentrated load at F)
$V_{FR} = 0$ kN

ΔV_F = concentrated load at F = 6 kN ∴ ok.

Calculations and/or Diagrams (cont'd):

3. Area Method : Shear "V"

$V_A = 0$ (Free end)

$V_{B_L} - V_A$ = area of uniform load $= -4(3) \Rightarrow V_{B_L} = -12 + 0 = -12$ KN

$V_{BR} = V_{BL} + y_B = -12 + 24$ KN $= 12$ KN

$V_C - V_{B_R}$ = area of $w = 0 \Rightarrow V_C = V_B = 12$ KN

$V_D - V_C$ = area of $w = -4(3) \Rightarrow V_D = 12 - 4(3) = 0$

$V_{E_L} - V_D$ = area of $w = -4(3) \Rightarrow V_{E_L} = 0 - 12 = -12$ KN

$V_{ER} = V_{ER} + y_E = -12 + 18 = 6$ KN

$V_{F_L} - V_{ER}$ = area of $w = 0 \Rightarrow V_F = V_{ER} = 6$ KN (concentrated load at F ∴ ok)

$V_{FR} = V_{FL} - 6$ KN $= 6$ KN $- 6$ KN $= 0$ ∴ (OK $\Sigma F_y = 0$)

* Moments "M"

$M_A = 0$ (Free end)

$M_B - M_A$ = area of S.F.D$_{A\to B}$ $= -\frac{1}{2} * 12 * 4 = -24 \Rightarrow M_B = 0 - 24 = -24$ KN.m

$M_C - M_B$ = area of S.F.D$_{B\to C}$ $\Rightarrow M_C = -24 + 12*2 = 0$

$M_{D_L} - M_C$ = area of S.F.D$_{C\to D}$ $\Rightarrow M_{DL} = \frac{1}{2} * 12 * 4 + 0 = 24$ KN.m

~~counterclockwise moment~~

$M_{DR} = M_{DL} - M_D \Rightarrow 24 - 24 = 0$

$M_E - M_{DR}$ = area of S.F.D$_{D\to E}$ $\Rightarrow M_E = 0 - \frac{1}{2} * 12 * 4 = -24$ KN.m

$M_F - M_E$ = area of S.F.D$_{E\to F}$ $\Rightarrow M_F = -24 + 6*4 = 0$ ∴ ok (Free end)

* $M_{max}^{+ve} = 24$ KN.m

* $M_{max}^{-ve} = -24$ KN.m

* $V_{max}^{+ve} = 12$ KN & $V_{max}^{-ve} = 12$ KN

EXTRA SHEET 1: Continued from page

Name: ____________ **ID#:** ____________ **Section:** ______

Calculations and/or Diagrams:

EXTRA SHEET 2: Continued from page ____

Name: ____________ **ID#:** ____________ **Section:** ____

Calculations and/or Diagrams: