QUIZ 1

Spring 2011-12

(March 28, 2012)

CIVE210 – STATICS

CLOSED BOOK, 1 HR 30 MN

Name: Solutions **ID#:** 007

NOTES

- 4 PROBLEMS– 12 PAGES.
- ALL YOUR ANSWERS SHOULD BE PROVIDED ON THE QUESTION SHEETS.
- **TWO EXTRA SHEETS ARE PROVIDED AT THE END.**
- **ASK FOR ADDITIONAL SHEETS IF YOU NEED MORE SPACE.**
- SOME ANSWERS MAY REQUIRE MUCH LESS THAN THE SPACE PROVIDED.
- ***DO NOT*** USE THE BACK OF THE SHEETS FOR ANSWERS.
- *DRAFT BOOKLET WILL BE PROVIDED; BUT DO NOT USE FOR ANSWERS.*
- BOTH QUESTION SHEETS AND DRAFT BOOKLET SHOULD BE RETURNED.
- CHECK BOXES ARE TO CONFIRM THAT YOU HAVE SOLVED A QUESTION. ☑

YOUR COMMENT(S)

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_ _

DO NOT WRITE IN THE SPACE BELOW

MY COMMENT(S)

_ _

_ _

YOUR GRADE

Problem I:	20 /20
Problem II:	20 /20
Problem III:	25 /25
Problem IV:	35 /35
TOTAL:	100 /100

Problem I: *(20 points)*

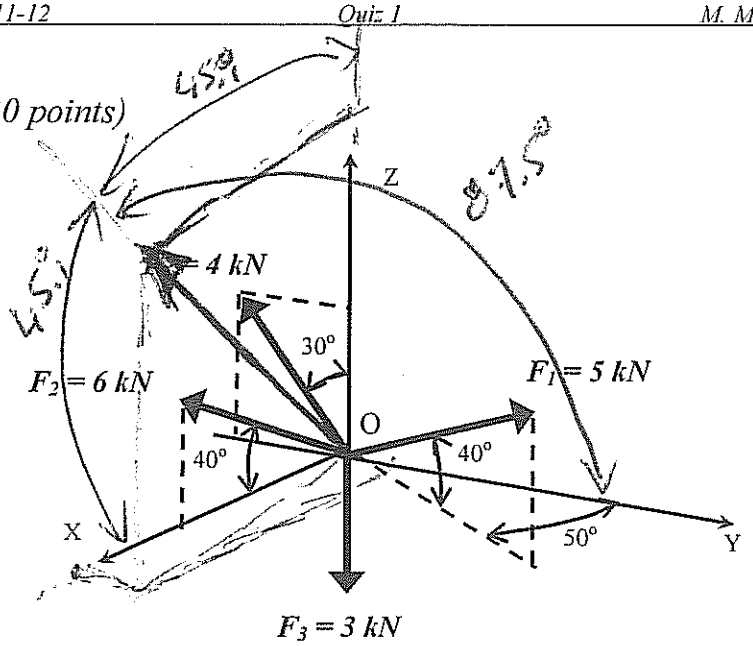


Figure I

Tick Boxes to check that you solved all questions

The four forces shown in Figure I are concurrent at point O. F_1 lies in a vertical plane that makes an angle of 50 degrees with the Y-axis; F_2 is in the X-Z plane; F_3 is a vertical force; and F_4 is in the Y-Z plane.

1. Compute the resultant force in Cartesian vector notation. *(13 points)*
2. Calculate the magnitude and direction of the resultant force, and sketch it on the figure above. *(7 points)*

Calculations and/or Diagrams:

1. $\vec{F_R} = \vec{F_1} + \vec{F_2} + \vec{F_3} + \vec{F_4}$

$\vec{F_1}$: $F_{1(xy\ plane)} = 5\cos 40° = 3.83\ kN \Rightarrow F_{1x} = 3.83 \sin 50 = 2.93$ kN; $F_{1y} = 3.83 \cos 50 = 2.46$ kN; $F_{1z} = 5 \sin 40° = 3.21$ kN

$\vec{F_1} = 2.93\vec{i} + 2.46\vec{j} + 3.21\vec{k}$

$\vec{F_2}$: $F_{2x} = 6\cos 40 = 4.60$ kN; $F_{2y} = 0$; $F_{2z} = 6 \sin 40 = 3.86$ kN

$\vec{F_2} = 4.60\vec{i} + 0\vec{j} + 3.86\vec{k}$

$\vec{F_3}$: $\vec{F_3} = -3\vec{k}$

$\vec{F_4}$: $F_{4x} = 0$; $F_{4y} = -4 \sin 30 = -2$ kN; $F_{4z} = 4\cos 30 = 3.46$ kN

$\vec{F_4} = 0\vec{i} - 2\vec{j} + 3.46\vec{k}$

2. $\vec{F_R} = 7.53\vec{i} + 0.46\vec{j} + 7.53\vec{k}$

$F_R = \sqrt{F_x^2 + F_y^2 + F_z^2} = 10.66$ kN

Direct cosines: $\cos\alpha = \cos\gamma = \frac{7.53}{10.66} \Rightarrow \alpha = \gamma = 45.1°$

$\cos\beta = \frac{0.46}{10.66} \Rightarrow \beta = 87.5°$

Problem II: *(20 points)*

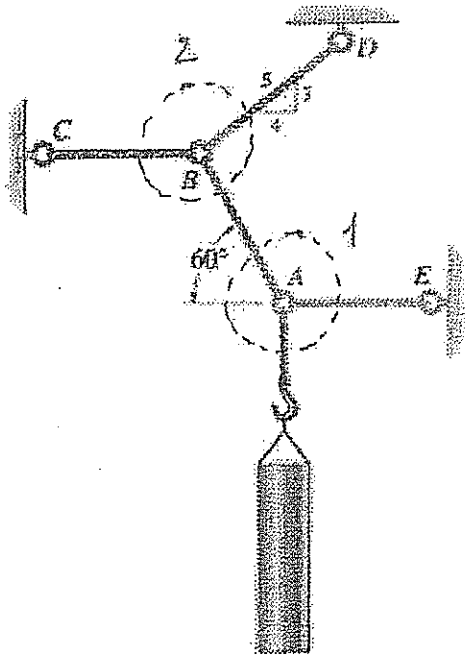


Figure II

The cables system suspends the cylinder in the equilibrium position shown in Figure II.

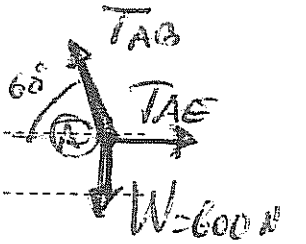
1. If the weight of the cylinder is 600 N, compute the tension forces in each of the cables. *(12 points)* ☐
2. Evaluate the system if the maximum tension that each of the cables can carry is 700 N (explain in 2-3 lines maximum). In this case, what is the maximum cylinder weight that the system can carry. *(8 points)* ☐

Calculations and/or Diagrams:

1. $\Sigma F_y = 0 \Rightarrow T_{AB} \sin 60 = 600$ — Joint A

$\Rightarrow T_{AB} = 692.82\ N$

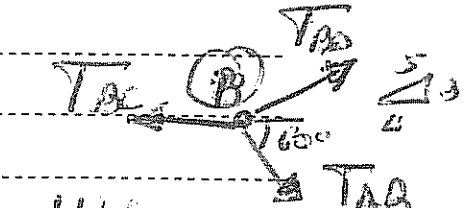
$\Sigma F_x = 0 \Rightarrow T_{AE} = T_{AB} \cos 60 = 346.41\ N$



$\Sigma F_y = 0 \Rightarrow T_{BD} \times \frac{3}{5} = T_{AB} \sin 60$

$\Rightarrow T_{BD} = 1000\ N$

$\Sigma F_x = 0 \Rightarrow T_{BC} = T_{AB} \cos 60 + T_{BD} \times \frac{4}{5} = 1146.41\ N$



2. $T_{max} = 700\ N < T_{BD} < T_{BC} \Rightarrow$ System will fail

Max Tension in BC = $W = 600\ N \rightarrow T_{BC} = 1146.41\ N$

$\Rightarrow W_{max} = 600 \times \frac{700}{1146.4} = 366.4\ N$ | $W_{max} = ? \rightarrow T_{BC} = T_{max} = 700\ N$

Problem III: *(25 points)*

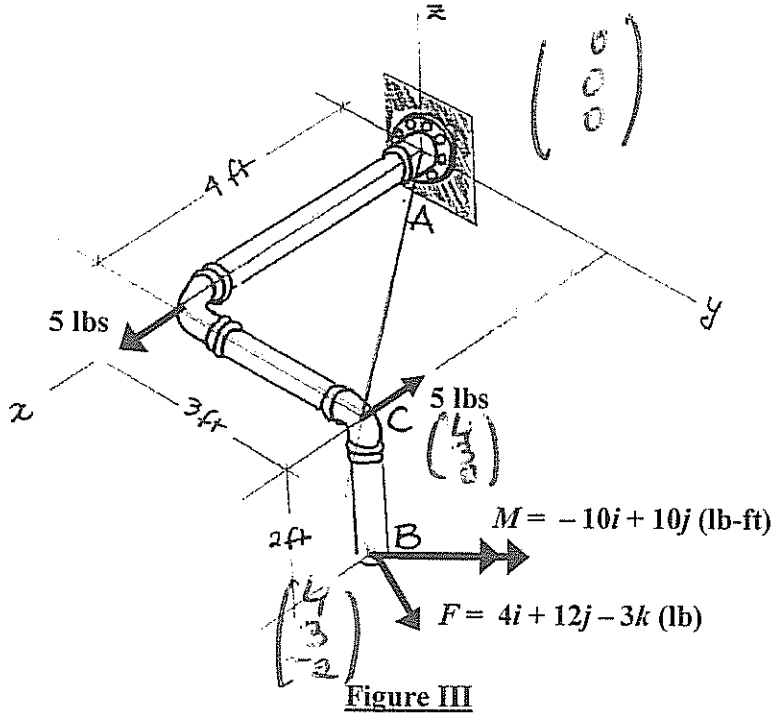


Figure III

The rigid pipe system is subjected to the forces and moments shown in Figure III.

1. Compute the resultant fore and moment system at the support A in Cartesian vector form. *(15 points)* ☐
2. Determine this resultant system about an axis extending between points A and C. Express the results as Cartesian vectors. *(10 points)* ☐

Calculations and/or Diagrams:

1. $\vec{F_A} = \vec{F} = 4\vec{i} + 12\vec{j} - 3\vec{k}$

$\vec{M_A} = \vec{M} + \vec{r}_{AB} \times \vec{F} + M_{couple}$

$\vec{r}_{AB} \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 4 & 3 & -2 \\ 4 & 12 & -3 \end{vmatrix} = 15\vec{i} + 4\vec{j} + 36\vec{k}$

$M_{couple} = 5 \times 3 = 15 \text{ lb.ft} = 15\vec{k}$

$\vec{M_A} = (-10+15)\vec{i} + (10+4)\vec{j} + (+36+15)\vec{k} = 5\vec{i} + 14\vec{j} + 51\vec{k}$

2. $\vec{u}_{AC} = \frac{\vec{AC}}{|\vec{AC}|} = \frac{4\vec{i} + 3\vec{j} + 0\vec{k}}{\sqrt{4^2 + 3^2 + 0}} = \frac{4}{5}\vec{i} + \frac{3}{5}\vec{j}$

$\vec{F}_{AC} = (\vec{F}_A \cdot \vec{u}_{AC})\,\vec{u}_{AC} = 10.4\,\vec{u}_{AC} = 8.32\vec{i} + 6.24\vec{j}$

$\vec{M}_{AC} = (\vec{M}_A \cdot \vec{u}_{AC})\,\vec{u}_{AC} = 12.4\,\vec{u}_{AC} = 9.92\vec{i} + 7.44\vec{j}$

Problem IV: *(35 points)*

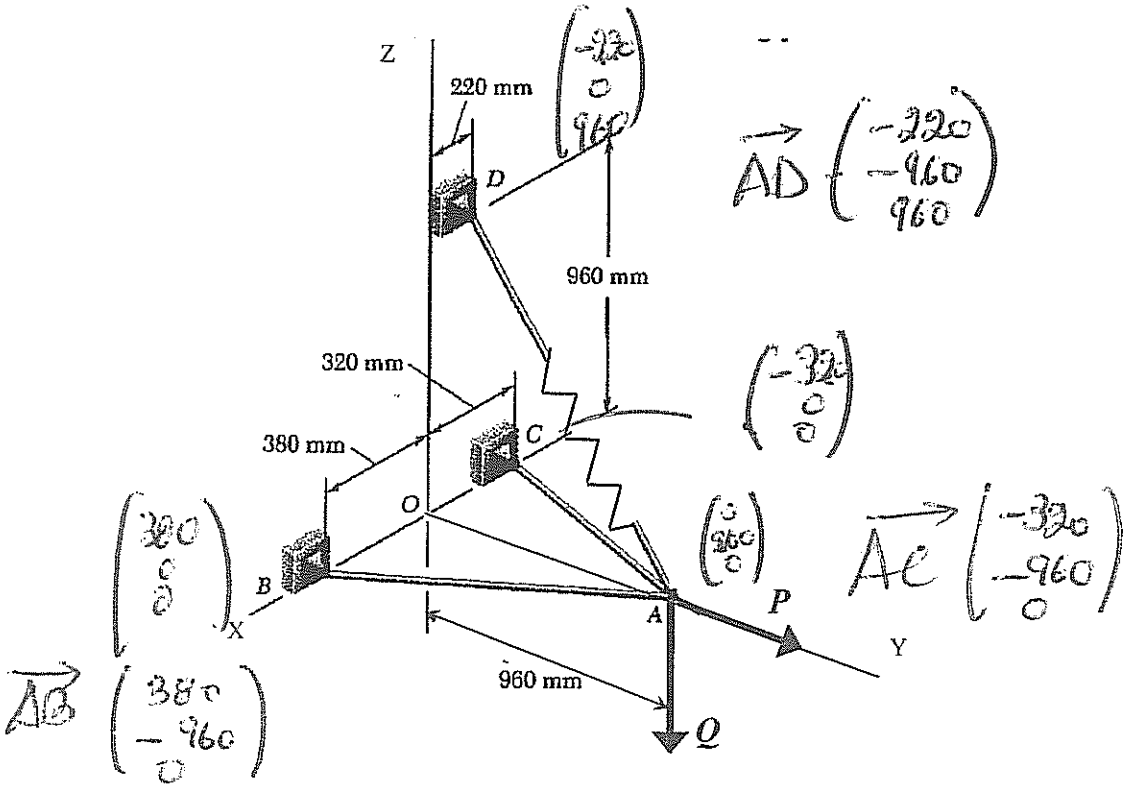


Figure IV

Two cables and a spring are connected at point A as shown in Figure IV. The spring has a stiffness of 4,000 N/m. Determine the ***maximum values*** of the forces ***P*** and ***Q*** that can be applied at A in the directions shown, if the maximum displacement that the spring can withstand before failing is 100 mm, and the maximum tension that each cable can support before failing is 500 N. (Hint: Start by maximizing ***Q***, and then move to maximizing ***P***).

Calculations and/or Diagrams:

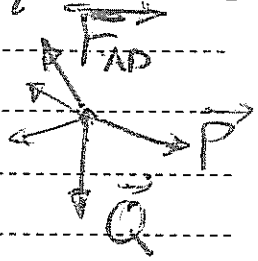
Equilibrium of joint A $\Rightarrow \sum \vec{F} = 0$ | $\sum F_x = 0$, $\sum F_y = 0$, $\sum F_z = 0$

Note: AB & AC in XY plane $\Rightarrow$ No component in Z

$\sum F_z = 0 \Rightarrow$ Relate F_{AD} to Q

$\vec{F}_{AD} = F_{AD} \cdot \vec{u}_{AD}$

$= \dfrac{-220\vec{i} - 960\vec{j} + 960\vec{k}}{\sqrt{220^2 + 960^2 + 960^2}} F_{AD} \rightarrow 1375.35$



$P_{max} \leftarrow F_{AD_{max}} = k_2 \Delta_{max} = \dfrac{4000\ N}{1000\ mm} \times 100\ mm = 400\ N$

$\vec{F}_{AD} = 63.98\vec{i} - 279.28\vec{j} + 279.28\vec{k}$

$\vec{Q} = -Q\vec{k}$

$\sum F_z = 0 \quad F_{AD} + Q = 0$

$\Rightarrow P_{max} = 279.2\ lb$ (N)

Calculations and/or Diagrams (cont'd):

$\vec{F}_{AB} = F_{AB}\,\vec{\mu}_{AB} = \dfrac{360\vec{i} - 960\vec{j}}{\sqrt{360^2 + 960^2}} F_{AB} = (0.36\vec{i} - 0.93\vec{j})F_{AB}$

$\vec{F}_{AC} = F_{AC}\,\vec{\mu}_{AC} = \dfrac{-320\vec{i} - 960\vec{j}}{\sqrt{320^2 + 960^2}} F_{AC} = (-0.316\vec{i} - 0.949\vec{j})F_{AC}$

F_{AB} or F_{AC} $max = 500$ N $\quad \sum\vec{F} = 0 \Rightarrow \begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \end{cases}$

$\vec{F}_{AB} + \vec{F}_{AC} + \vec{F}_{AD} + \vec{W} + \vec{P} = 0$

X Try $F_{AC} = 500$ N $\quad \sum F_x = 0$

$\Rightarrow +0.36F_{AB} - 158.1 - 63.98 = 0$

$\Rightarrow F_{AB} = 603.5$ N > 500 N not OK

✓ Try $F_{AB} = 500$ N $\quad \sum F_x = 0$

$\Rightarrow +180 - 0.316F_{AC} - 63.98 = 0$

$\Rightarrow F_{AC} = 379.8 < 500$ N OK

$\sum F_y = 0$

$\Rightarrow -0.93 \cdot 500 - 0.949 \times 379.6 - 279.2 + P = 0$

$\Rightarrow P = 1104.7$ N