QUIZ 1

Fall 2012-13

(November 5, 2012)

CIVE210 – STATICS

CLOSED BOOK, 1 HR 30 MN

Name: Solution **ID#:** CC 7

NOTES

- 4 PROBLEMS– 13 PAGES.
- ALL YOUR ANSWERS SHOULD BE PROVIDED ON THE QUESTION SHEETS.
- **TWO EXTRA SHEETS ARE PROVIDED AT THE END.**
- **ASK FOR ADDITIONAL SHEETS IF YOU NEED MORE SPACE.**
- SOME ANSWERS MAY REQUIRE MUCH LESS THAN THE SPACE PROVIDED.
- *DO NOT* USE THE BACK OF THE SHEETS FOR ANSWERS.
- *DRAFT BOOKLET WILL BE PROVIDED; BUT DO NOT USE FOR ANSWERS.*
- BOTH QUESTION SHEETS AND DRAFT BOOKLET SHOULD BE RETURNED.
- CHECK BOXES ARE TO CONFIRM THAT YOU HAVE SOLVED A QUESTION. ☑

YOUR COMMENT(S)

DO NOT WRITE IN THE SPACE BELOW

MY COMMENT(S)

YOUR GRADE

Problem I:	15 /15
Problem II:	30 /30
Problem III:	25 /25
Problem IV:	30 /30
Bonus/Extras – Organization, Neatness, Special, ...:	---
TOTAL:	100 /100

Problem I: *(15 points)*

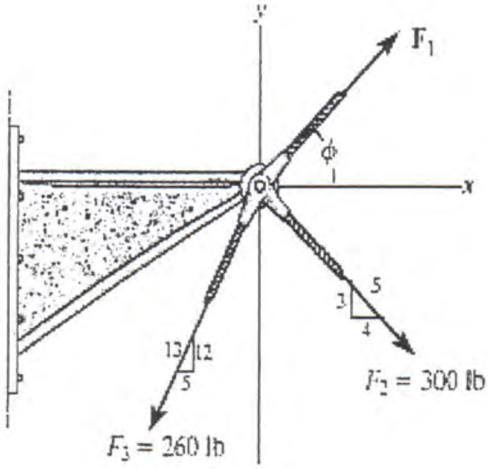


Figure I

Tick Boxes to check that you solved all questions

Referring to Figure I, if the magnitude of the resultant force acting on the bracket is 400 lb directed along the positive *x* axis, determine the magnitude of $\mathbf{F_1}$ and its direction ϕ. *(15 points)*

Calculations and/or Diagrams:

Components F_x x F_y =

$F_{1x} = F_1 \cos\phi$ $\quad F_{1y} = F_1 \sin\phi$

$F_{2x} = 300 \times 4/5 = 240$ lb $\quad F_{2y} = -300 \times 3/5 = -180$ lb

$F_{3x} = -260 \times 5/13 = -100$ lb $\quad F_{3y} = -260 \times \frac{12}{13} = -240$ lb

$F_{Rx} = +400$ lb $\quad F_{Ry} = 0$

$F_{Rx} = \Sigma F_x \; (\xrightarrow{+}) \Rightarrow 400 = F_1 \cos\phi + 240 - 100$

$\Rightarrow F_1 \cos\phi = 260 \quad (1)$

$F_{Ry} = \Sigma F_y \; (+\uparrow) \Rightarrow 0 = F_1 \sin\phi - 180 - 240$

$\Rightarrow F_1 \sin\phi = 420 \quad (2)$

$\frac{(2)}{(1)} \Rightarrow \tan\phi = \frac{420}{260} = \frac{21}{13} \Rightarrow \boxed{\phi = 58.2°} \Rightarrow \boxed{F_1 = 494 \text{ lbs}}$

Problem II: *(30 points)*

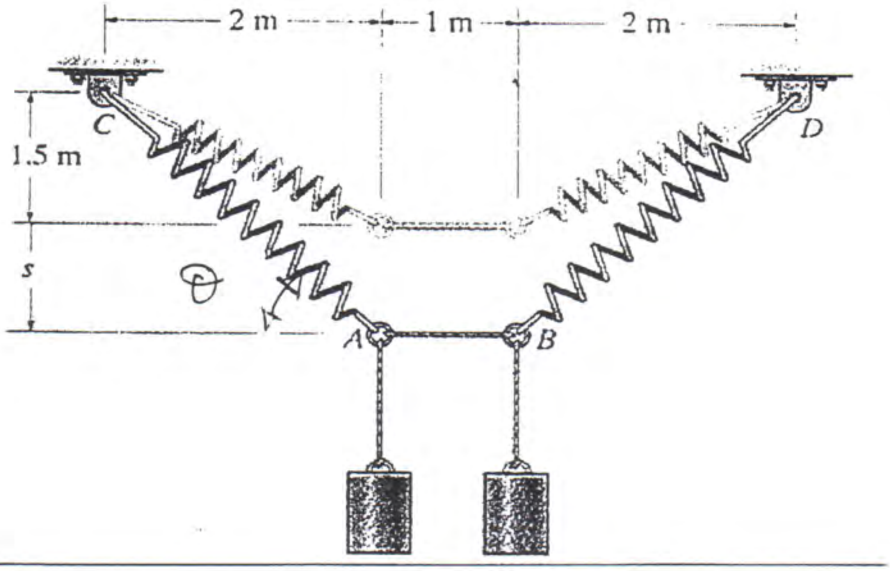


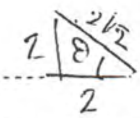
Figure II

1- Referring to Figure II, the identical cylinders weighing 20 N each cause a sag of $s = 0.5$m in the system when suspended from the rings at A and B. Determine the stiffness k of the identical springs. Note that $s = 0$ when the cylinders are removed. *(15 points)*

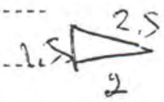
2- Using the stiffness k obtained earlier, if the cylinder weights are now 40 N each, compute the new sag s. Compare with $s = 0.5$m and very briefly comment (1 or 2 lines). *(15 points)*

Calculations and/or Diagrams:

1. $S = 0.5m \Rightarrow \theta = 45°$



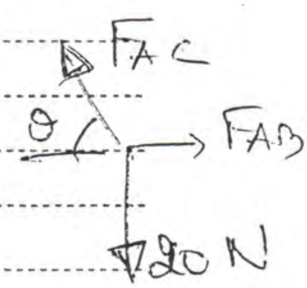
l spring initial $= l_0 = \sqrt{2^2 + 1.5^2} = 2.5m$



l spring final $= l = 2\sqrt{2} = 2.828m$

$\Delta l = 2.828 - 2.5 = 0.328m$

$F_{AC} = k \times 0.328$ (N)



$\Sigma F_y = 0$ (↑+) $\Rightarrow F_{AC} \sin\theta = 20$

$\Rightarrow k \times 0.328 \times \frac{\sqrt{2}}{2} = 20$

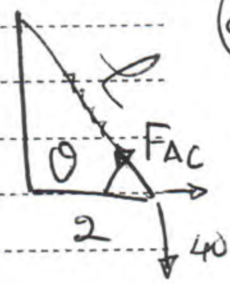
$\Rightarrow k = 86.25$ N/m

Calculations and/or Diagrams (cont'd):

2. Using $k = 86.25\,N/m$ $W = 40N$

$(d = 1.5 + S)$

$\sum F_y = 0$ $F_{AC} \sin\theta = 40$ ①



$\ell = \sqrt{d^2 + 2^2} \Rightarrow d = \sqrt{\ell^2 - 4}$

$\sin\theta = \dfrac{d}{\sqrt{d^2+4}}$ (in d) $= \dfrac{d}{\ell} = \dfrac{\sqrt{\ell^2-4}}{\ell}$ (in ℓ)

Solve in ℓ

$F_{AC} = k\,\Delta\ell = 86.25(\ell - \ell_0)$ ② $\ell_0 = 2.5m$ (initial)

$\Rightarrow ② \to ① \Rightarrow 86.25(\ell - 2.5) \times \dfrac{\sqrt{\ell^2-4}}{\ell} - 40 = 0$

Solve for ℓ ⇒ Try ⟹ $\ell = 3.106\,m$

$d = \sqrt{\ell^2 - 4} = 2.38m = 1.5 + S \Rightarrow \boxed{S = 0.88m}$

OR

$F_{AC} \sin\theta = 40$ $F_{AC} = k\Delta\ell = k\left(\sqrt{d^2+4} - 2.5\right)$

$\Rightarrow \sin\theta = \dfrac{40}{F_{AC}}$ ①

& $\sin\theta = \dfrac{d}{\sqrt{d^2+4}}$ ②

Solve in d

$① = ② \Rightarrow \dfrac{d}{\sqrt{d^2+4}} = \dfrac{40}{86.25\left(\sqrt{d^2+4} - 2.5\right)}$

Try $d = 2.37 \Rightarrow \boxed{S = 0.87\,m}$

Comment: Load double but Sag did not double

⟹ not linear relation

[Single Spring k $\updownarrow S$ $\downarrow F$ $F = kS \Rightarrow$ linear)

Problem III: *(25 points)*

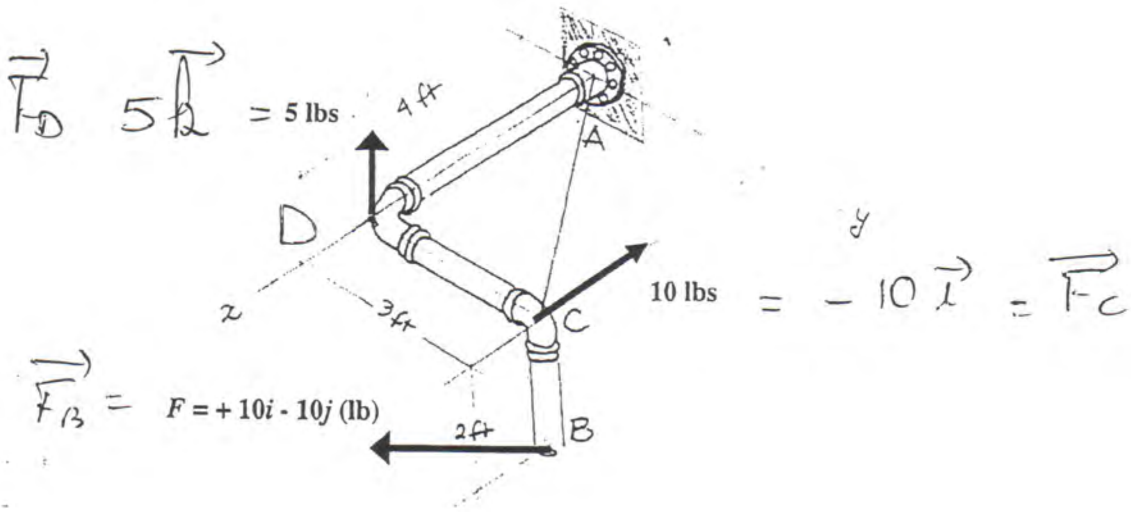


Figure III

The rigid pipe system is subjected to the forces shown in Figure III.

1. Use a cross-product approach, compute the moment from the three forces at the support A in Cartesian vector form. *(12 points)*.
2. Re-compute the three components Mx, My, and Mz at A due to the three forces using a simple scalar approach, and compare with question 1. *(6 points)*
3. Determine the component of this moment about an axis extending between points A and C. Express the results as Cartesian vectors. *(7 points)*

Calculations and/or Diagrams:

$A\begin{pmatrix}0\\0\\0\end{pmatrix}$ $B\begin{pmatrix}4\\3\\-2\end{pmatrix}$ $C\begin{pmatrix}4\\3\\0\end{pmatrix}$ $D\begin{pmatrix}4\\0\\0\end{pmatrix}$

$\left(\vec{r}_{AB} = \vec{AB}\right) = \begin{pmatrix}4\\3\\-2\end{pmatrix}$ $\vec{AC}\begin{pmatrix}4\\3\\0\end{pmatrix}$ $\vec{AD}\begin{pmatrix}4\\0\\0\end{pmatrix}$

$\vec{M}_A = \vec{AB} \times \vec{F}_B + \vec{AC} \times \vec{F}_C + \vec{AD} \times \vec{F}_D$

$= \begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\4 & 3 & -2\\10 & -10 & 0\end{vmatrix} + \begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\4 & 3 & 0\\-10 & 0 & 0\end{vmatrix} + \begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\4 & 0 & 0\\0 & 0 & 5\end{vmatrix}$

$= \left(-20\vec{i} - 20\vec{j} - 70\vec{k}\right) + \left(30\vec{k}\right) + \left(-20\vec{j}\right)$

$\vec{M}_A = \underset{M_x}{-20\vec{i}} \underset{M_y}{-40\vec{j}} \underset{M_z}{-40\vec{k}}$ or $M_x = -20$ lb.ft, $M_y = -40$ lb.ft, $M_z = -40$ lb.ft

Calculations and/or Diagrams (cont'd): F_B F_E F_D

2. $M_x = -10\times 2 \quad 0 \quad 0 = -20\ \text{k.ft}$

$M_y = -10\times 2 \quad 0 \quad -5\times 4 = -40\ \text{k.ft}$

$M_z = -10\times 3 - 10\times 4 + 10\times 3 \quad 0 = -40\ \text{k.ft}$

$\Rightarrow \vec{M}_A = -20\vec{i} - 40\vec{j} - 20\vec{k}$ Same as Q1.

3. $\vec{AC} = \begin{pmatrix} 4 \\ 3 \\ 0 \end{pmatrix} \qquad |\vec{AC}| = \sqrt{4^2 + 3^2 + 0^2} = 5$

Unit $\vec{\mu}_{AC} = \frac{4}{5}\vec{i} + \frac{3}{5}\vec{j}$

$\vec{M}_{AC} = \left(\vec{M}_A \cdot \vec{\mu}_{AC}\right) \vec{\mu}_{AC}$

$= \left(-\frac{20\times 4}{5} - 40\times\frac{3}{5}\right)\left(\frac{4}{5}\vec{i} + \frac{3}{5}\vec{j}\right)$

$= -32\vec{i} - 24\vec{j}$

Problem IV: *(30 points)*

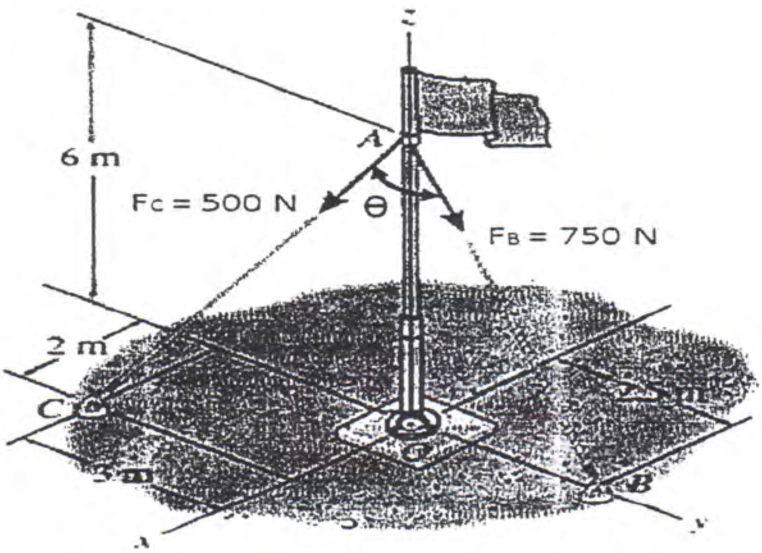


Figure IV

The two forces F_C and F_B are acting on the pole at point A as shown in Figure IV.

1. Determine the projection of the resultant force F_R of F_C and F_B acting along CB and perpendicular to it. Write the results in vector Cartesian form. *(20 points)*
2. Determine the angle θ between the two forces. *(10 points)*

Calculations and/or Diagrams:

$A\begin{pmatrix}0\\0\\6\end{pmatrix}$ $B\begin{pmatrix}0\\2.5\\0\end{pmatrix}$ $C\begin{pmatrix}2\\-3\\0\end{pmatrix}$

1. $\overrightarrow{AB} = \begin{pmatrix}0\\2.5\\-6\end{pmatrix}$ $|\overrightarrow{AB}| = \sqrt{2.5^2+6^2} = 6.5\,m$

$\overrightarrow{AC}\begin{pmatrix}2\\-3\\-6\end{pmatrix}$ $|\overrightarrow{AC}| = \sqrt{2^2+3^2+6^2} = 7\,m$

$\overrightarrow{u_{AB}} = \frac{5}{13}\vec{j} - \frac{12}{13}\vec{k}$ $\overrightarrow{F_{AB}} = 750\,\overrightarrow{u_{AB}} = 288.46\,\vec{j} - 692.31\,\vec{k}$

$\overrightarrow{u_{AC}} = \frac{2}{7}\vec{i} - \frac{3}{7}\vec{j} - \frac{6}{7}\vec{k}$ $\overrightarrow{F_{AC}} = 500\,\overrightarrow{u_{AC}} = 142.85\,\vec{i} - 214.29\,\vec{j} - 428.57\,\vec{k}$

Calculations and/or Diagrams (cont'd):

$\vec{F_R} = \vec{F_B} + \vec{F_C} = 142.85\vec{i} + 76.17\vec{j} - 1120.07\vec{k}$

$\vec{CB} = \begin{pmatrix} -2 \\ 5.5 \\ 0 \end{pmatrix}$ $|\vec{CB}| = \sqrt{2^2 + 5.5^2} = 5.852\,m$

$\vec{u_{CB}} = -0.342\vec{i} + 0.940\vec{j}$

Proj $\vec{F_R}/_{CB} = (\vec{F_R} \cdot \vec{u_{CB}})\,\vec{u_{CB}} = 20.865\,\vec{u_{CB}}$

$= -7.14\vec{i} + 19.61\vec{j}$

1) $F_{\perp BC} = \vec{F_B} - \vec{F_R}/_{\vec{CB}} = 150\vec{i} + 54.56\vec{j} - 1120.$

9. Dot Product: $\cos\theta = \dfrac{\vec{F_B} \cdot \vec{F_C}}{|\vec{F_B}|\,|\vec{F_C}|} =$

$\Rightarrow \cos\theta = 0.6263 \Rightarrow \theta = 51.22°$

or Cosine law: $F_R^2 = F_B^2 + F_C^2 + 2F_B F_C \cos\theta$

$\Rightarrow \cos\theta = 0.6263$ $\theta = 51.22°$