Homework # 1 - SOLUTION

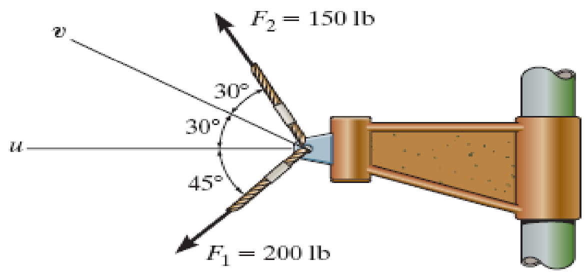
CIVE210 – STATICS

Topics: Vectors and Forces (Chapter 2)

Textbook: Engineering Mechanics, by R.C. Hibbeler
Pearson, 12th Edition

Problems:
Chapter 2: Problems 2-4, 2-6, 2-15, 2-20, 2-24 (Use Parallelogram Law only)
Problems 2-32, 2-44, 2-53 (Use 2-D Cartesian Vector Notation)
Problems 2-68, 2-77, 2-87, 2-104, 2-112, 2-121 (Use 3-D Cartesian Vector Notation and Dot Product)

2–4. Determine the magnitude of the resultant force acting on the bracket and its direction measured counterclockwise from the positive *u* axis.



The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*, respectively.

Applying the law of cosines to Fig. *b*,

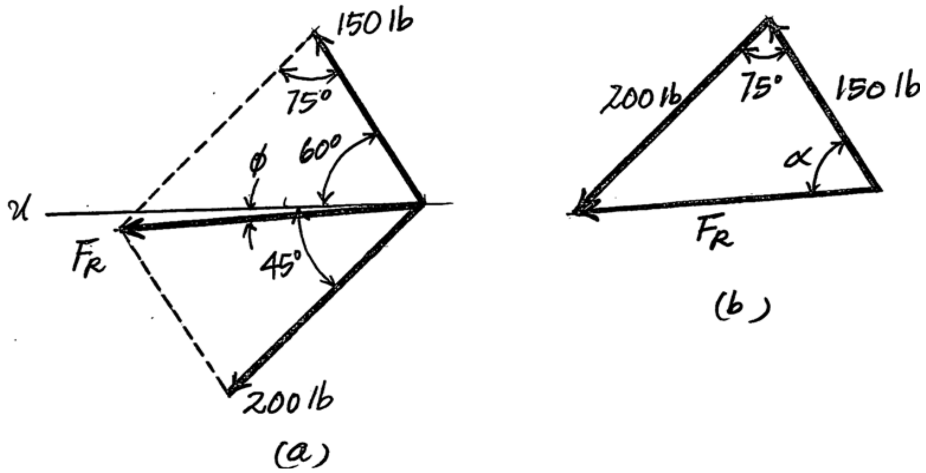
$$F_R = \sqrt{200^2 + 150^2 - 2(200)(150)\cos 75°}$$
$$= 216.72 \text{ lb} = 217 \text{ lb}$$ **Ans.**

Applying the law of sines to Fig. *b* and using this result yields

$$\frac{\sin\alpha}{200} = \frac{\sin 75°}{216.72} \qquad \alpha = 63.05°$$

Thus, the direction angle ϕ of $\mathbf{F}_R$, measured counterclockwise from the positive *u* axis, is

$$\phi = \alpha - 60° = 63.05° - 60° = 3.05°$$ **Ans.**



2–6. Resolve $\mathbf{F}_2$ into components along the *u* and axes, and determine the magnitudes of these components.

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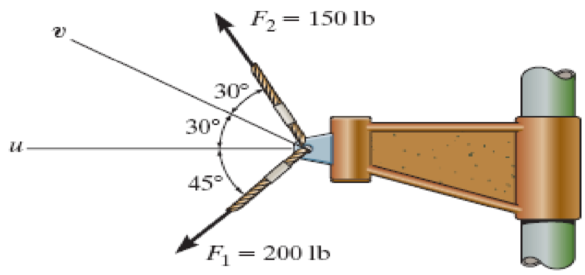
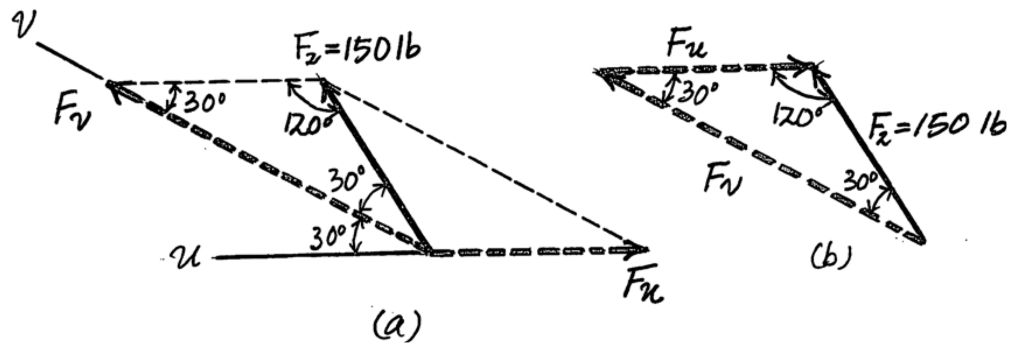


The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*, respectively.

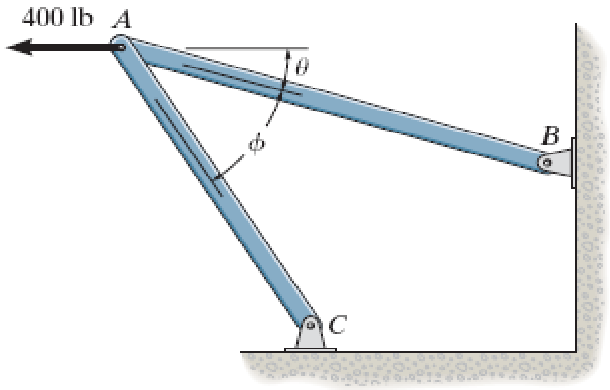
Applying the law of sines to Fig. *b*,

$$\frac{F_u}{\sin 30°} = \frac{150}{\sin 30°} \qquad F_u = 150 \text{ lb} \qquad \textbf{Ans.}$$

$$\frac{F_v}{\sin 120°} = \frac{150}{\sin 30°} \qquad F_v = 260 \text{ lb} \qquad \textbf{Ans.}$$



2–15. Determine the design angle between struts *AB* and *AC* so that the 400-lb horizontal force has a component of 600 lb which acts up to the left, in the same direction as from *B* towards *A*. Take θ = 30°.



Parallelogram Law : The parallelogram law of addition is shown in Fig. (a).

Trigonometry : Using law of cosines [Fig. (b)], we have

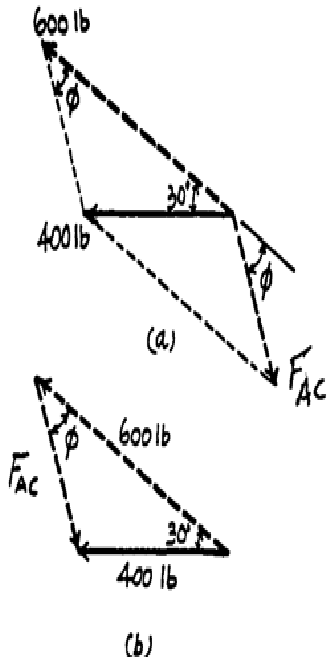
$$F_{AC} = \sqrt{400^2 + 600^2 - 2(400)(600)\cos 30^\circ} = 322.97 \text{ lb}$$

The angle ϕ can be determined using law of sines [Fig. (b)].

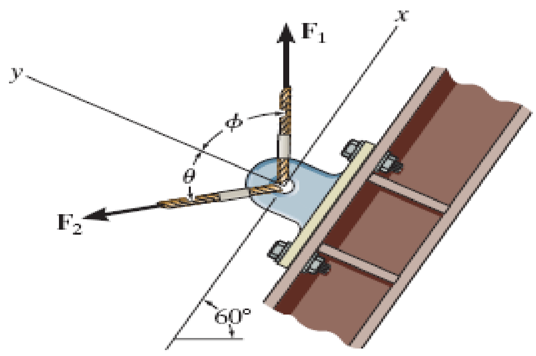
$$\frac{\sin\phi}{400} = \frac{\sin 30^\circ}{322.97}$$

$$\sin\phi = 0.6193$$

$$\phi = 38.3^\circ \qquad \text{Ans}$$



2–20. If $\phi = 45°$, $\mathbf{F_1} = 5$ kN, and the resultant force is 6 kN directed along the positive *y* axis, determine the required magnitude of $\mathbf{F}_2$ and its direction θ.



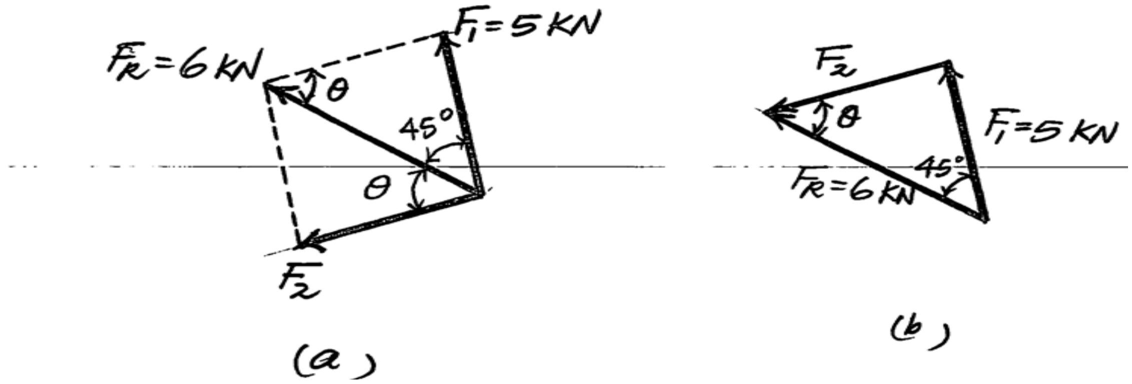
The parallelogram law of addition and triangular rule are shown in Figs. *a* and *b*, respectively.

Applying the law of cosines to Fig. *b*,

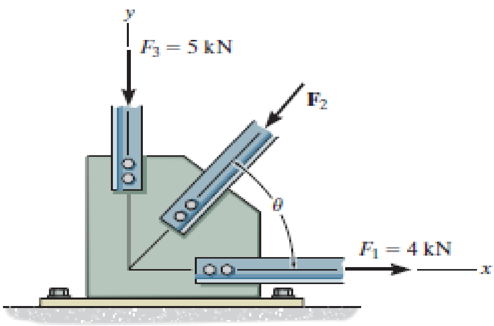
$$F_2 = \sqrt{6^2 + 5^2 - 2(6)(5)\cos 45°}$$
$$= 4.310 \text{ kN} = 4.31 \text{ kN} \qquad \textbf{Ans.}$$

Using this result and applying the law of sines to Fig. *b*, yields

$$\frac{\sin\theta}{5} = \frac{\sin 45°}{4.310} \qquad \theta = 55.1° \quad \textbf{Ans.}$$



2–24. If the resultant force $\mathbf{F_R}$ is directed along a line measured 75° clockwise from the positive *x* axis and the magnitude of $\mathbf{F_2}$ is to be a minimum, determine the magnitudes of $\mathbf{F_R}$ and $\mathbf{F_2}$ and the angle $\theta \le 90°$.



This problem can be solved by adding the forces successively, using the parallelogram law of addition, shown in Fig. *a*. Two triangular force diagrams, shown in Figs. *b* and *c*, can be derived from the parallelograms. For $\mathbf{F_1}$ to be minimum, it must be perpendicular to the resultant force' s line of action. Thus,

$$\theta = 90° - 75° = 15°$$ **Ans.**

Referring to Fig. *b*, F' and α can be determined.

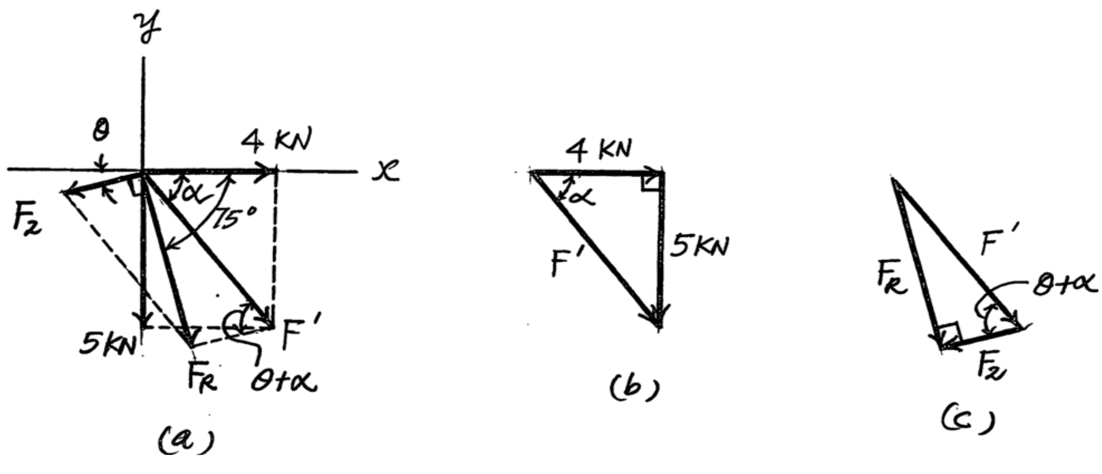
$$F' = \sqrt{4^2 + 5^2} = 6.403 \text{ kN}$$

$$\tan\alpha = \frac{5}{4} \qquad \alpha = 51.34°$$

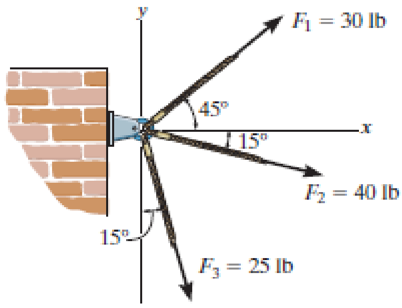
Using the results for θ, α, and F', $\mathbf{F}_R$ and $\mathbf{F}_2$ can be determined by referring to Fig. *c*.

$$F_2 = 6.403\cos(15° + 51.43°) = 2.57 \text{ kN}$$ **Ans.**

$$F_R = 6.403\sin(15° + 51.43°) = 5.86 \text{ kN}$$ **Ans.**



2–32. Determine the magnitude of the resultant force acting on the pin and its direction measured clockwise from the positive x axis.



Rectangular Components: By referring to Fig. a, the x and y components of $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ can be written as

$$(F_1)_x = 30\cos 45° = 21.21 \text{ lb} \qquad (F_1)_y = 30\sin 45° = 21.21 \text{ lb}$$
$$(F_2)_x = 40\cos 15° = 38.64 \text{ lb} \qquad (F_2)_y = 40\sin 15° = 10.35 \text{ lb}$$
$$(F_3)_x = 25\sin 15° = 6.47 \text{ lb} \qquad (F_3)_y = 25\cos 15° = 24.15 \text{ lb}$$

Resultant Force: Summing the force components algebraically along the x and y axes,

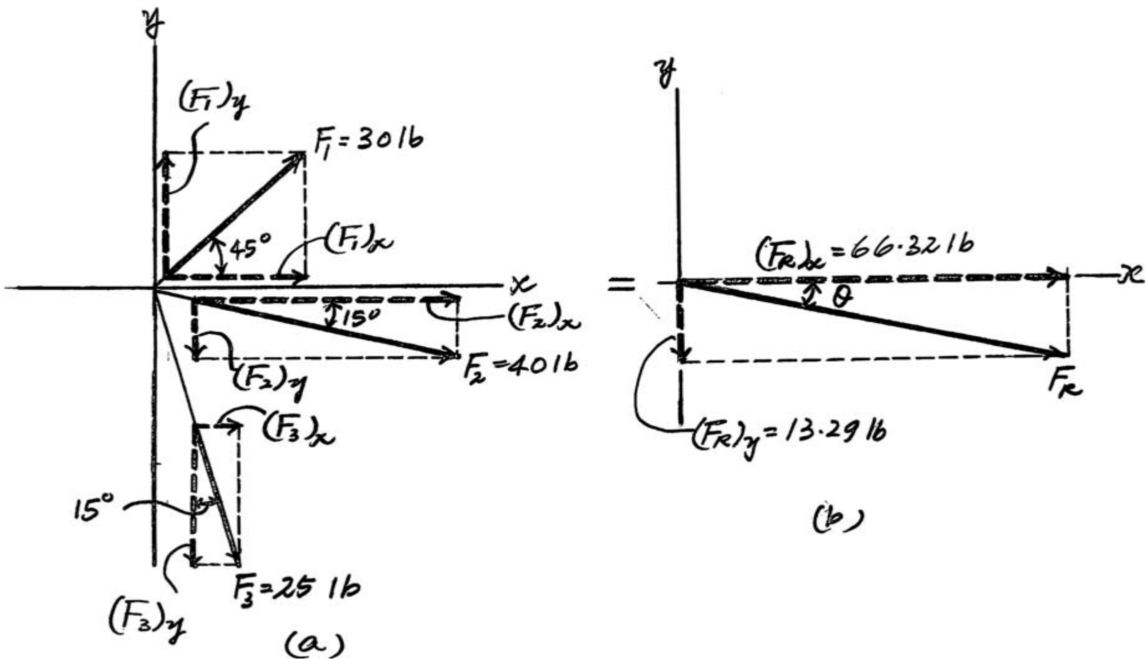
$$\overset{+}{\rightarrow}\Sigma(F_R)_x = \Sigma F_x; \quad (F_R)_x = 21.21 + 38.64 + 6.47 = 66.32 \text{ lb} \rightarrow$$
$$+\uparrow \Sigma(F_R)_y = \Sigma F_y; \quad (F_R)_y = 21.21 - 10.35 - 24.15 = -13.29 \text{ lb} = 13.29 \text{ lb} \downarrow$$

The magnitude of the resultant force $\mathbf{F}_R$ is

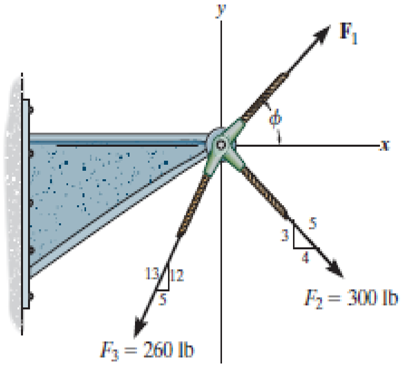
$$F_R = \sqrt{(F_R)_x{}^2 + (F_R)_y{}^2} = \sqrt{66.32^2 + 13.29^2} = 67.6 \text{ lb} \qquad \textbf{Ans.}$$

The direction angle θ of $\mathbf{F}_R$, measured clockwise from the positive x axis, is

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{13.29}{66.32}\right) = 11.3° \qquad \textbf{Ans.}$$



2–44. If the magnitude of the resultant force acting on the bracket is 400 lb directed along the positive x axis, determine the magnitude of $\mathbf{F_1}$ and its direction ϕ.



Rectangular Components: By referring to Fig. a, the x and y components of $\mathbf{F}_1$, $\mathbf{F}_2$, $\mathbf{F}_3$, and $\mathbf{F}_R$ can be written as

$$(F_1)_x = F_1\cos\phi \qquad (F_1)_y = F_1\sin\phi$$

$$(F_2)_x = 300\left(\frac{4}{5}\right) = 240 \text{ lb} \qquad (F_2)_y = 300\left(\frac{3}{5}\right) = 180 \text{ lb}$$

$$(F_3)_x = 260\left(\frac{5}{13}\right) = 100 \text{ lb} \qquad (F_3)_y = 260\left(\frac{12}{13}\right) = 240 \text{ lb}$$

$$(F_R)_x = 400 \text{ lb} \qquad (F_R)_y = 0$$

Resultant Force: Summing the force components algebraically along the x and y axes,

$$\overset{+}{\rightarrow}\Sigma(F_R)_x = \Sigma F_x; \quad 400 = F_1\cos\phi + 240 - 100$$

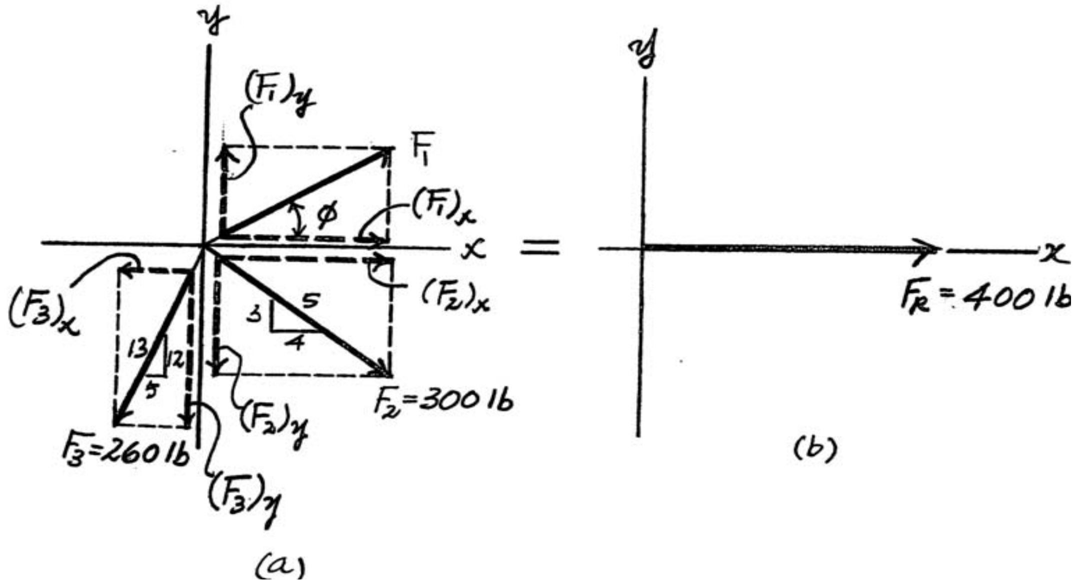
$$F_1\cos\phi = 260 \qquad (1)$$

$$+\uparrow\Sigma(F_R)_y = \Sigma F_y; \quad 0 = F_1\sin\phi - 180 - 240$$

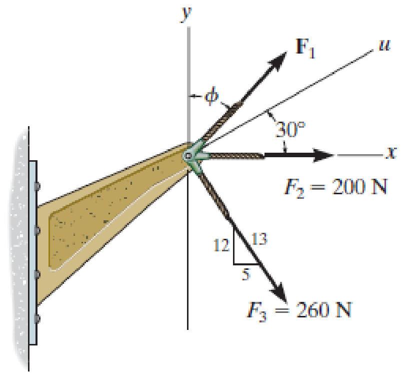
$$F_1\sin\phi = 420 \qquad (2)$$

Solving Eqs. (1) and (2), yields

$\phi = 58.2°$ $\qquad F_1 = 494$ lb **Ans.**

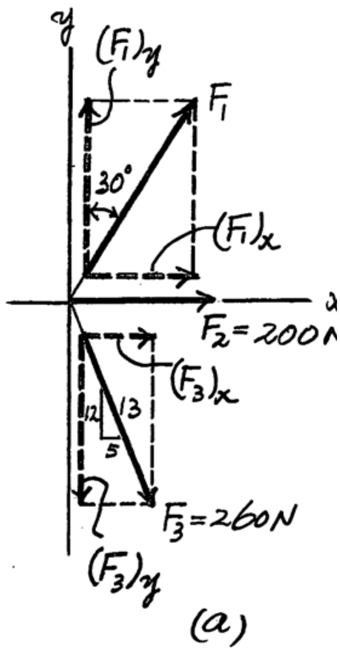


2–53. If the resultant force acting on the bracket is required to be a minimum, determine the magnitudes of $\mathbf{F}_1$, and the resultant force. Set $\phi = 30°$.



Rectangular Components: By referring to Fig. *a*, the *x* and *y* components of $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ can be written as

$$(F_1)_x = F_1 \sin 30° = 0.5F_1 \qquad (F_1)_y = F_1 \cos 30° = 0.8660F_1$$
$$(F_2)_x = 200\text{ N} \qquad (F_2)_y = 0$$
$$(F_3)_x = 260\left(\frac{5}{13}\right) = 100\text{ N} \qquad (F_3)_y = 260\left(\frac{12}{13}\right) = 240\text{ N}$$



Resultant Force: Summing the force components algebraically along the *x* and *y* axes,

$$\overset{+}{\rightarrow}\Sigma(F_R)_x = \Sigma F_x; \quad (F_R)_x = 0.5F_1 + 200 + 100 = 0.5F_1 + 300$$
$$+\uparrow \Sigma(F_R)_y = \Sigma F_y; \quad (F_R)_y = 0.8660F_1 - 240$$

The magnitude of the resultant force $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x^{\,2} + (F_R)_y^{\,2}} = \sqrt{(0.5F_1 + 300)^2 + (0.8660F_1 - 240)^2} = \sqrt{F_1^{\,2} - 115.69F_1 + 147600} \qquad (1)$$

Thus,

$$F_R^{\,2} = F_1^{\,2} - 115.69F_1 + 147\,600 \qquad (2)$$

The first derivative of Eq. (2) is

$$2F_R \frac{dF_R}{dF_1} = 2F_1 - 115.69 \qquad (3)$$

and the second derivative of Eq. (1) is

$$F_R \frac{d^2F_R}{dF_1^{\,2}} + \frac{dF_R}{dF_1}\frac{dF_R}{dF_1} = 1 \qquad (4)$$

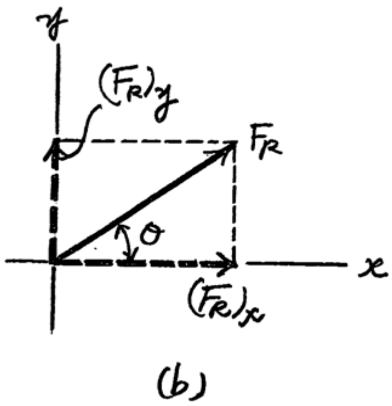
For $\mathbf{F}_R$ to be minimum, $\frac{dF_R}{dF_1} = 0$. Thus, from Eq. (3)

$$2F_R \frac{dF_R}{dF_1} = 2F_1 - 115.69 = 0$$

$$F_1 = 57.84\text{ N} = 57.8\text{ N}$$ **Ans.**

Substituting $F_1 = 57.84$ N and $\frac{dF_R}{dF_1} = 0$ into Eq. (4),

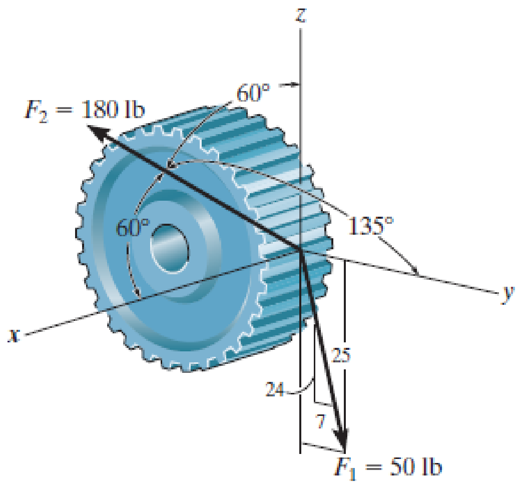
$$\frac{d^2F_R}{dF_1^{\,2}} = 0.00263 > 0$$



Thus, $F_1 = 57.84$ N produces a minimum $\mathbf{F}_R$. From Eq. (1),

$$F_R = \sqrt{(57.84)^2 - 115.69(57.84) + 147600} = 380\text{ N}$$ **Ans.**

2–68. The spur gear is subjected to the two forces caused by contact with other gears. Determine the resultant of the two forces and express the result as a Cartesian vector.



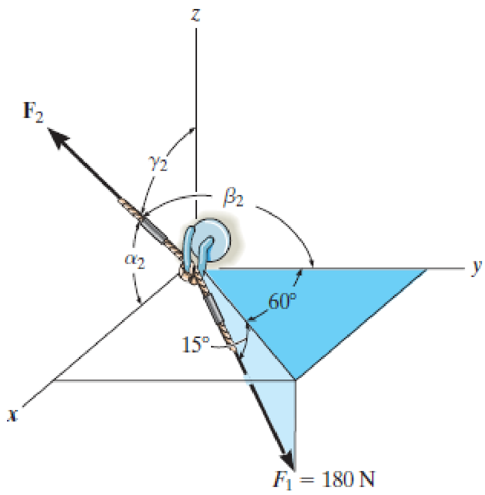
$$F_{Rx} = 180 \cos 60° = 90$$

$$F_{Ry} = \frac{7}{25}(50) + 180 \cos 135° = -113$$

$$F_{Rz} = -\frac{24}{25}(50) + 180 \cos 60° = 42$$

$$\mathbf{F}_R = \{90\mathbf{i} - 113\mathbf{j} + 42\mathbf{k}\} \text{ lb}$$ **Ans**

2–77. Determine the magnitude and coordinate direction angles of $\mathbf{F}_2$ so that the resultant of the two forces is zero.



$$\mathbf{F}_1 = (180\cos 15°)\sin 60°\,\mathbf{i} + (180\cos 15°)\cos 60°\,\mathbf{j} - 180\sin 15°\,\mathbf{k}$$

$$= 150.57\,\mathbf{i} + 86.93\,\mathbf{j} - 46.59\,\mathbf{k}$$

$$\mathbf{F}_2 = F_2\cos\alpha_2\,\mathbf{i} + F_2\cos\beta_2\,\mathbf{j} + F_2\cos\gamma_2\,\mathbf{k}$$

$$\mathbf{F}_R = \mathbf{0}$$

i components :

$$0 = 150.57 + F_2\cos\alpha_2$$

$$F_2\cos\alpha_2 = -150.57$$

j components :

$$0 = 86.93 + F_2\cos\beta_2$$

$$F_2\cos\beta_2 = -86.93$$

k components :

$$0 = -46.59 + F_2\cos\gamma_2$$

$$F_2\cos\gamma_2 = 46.59$$

$$F_2 = \sqrt{(-150.57)^2 + (-86.93)^2 + (46.59)^2}$$

Solving,

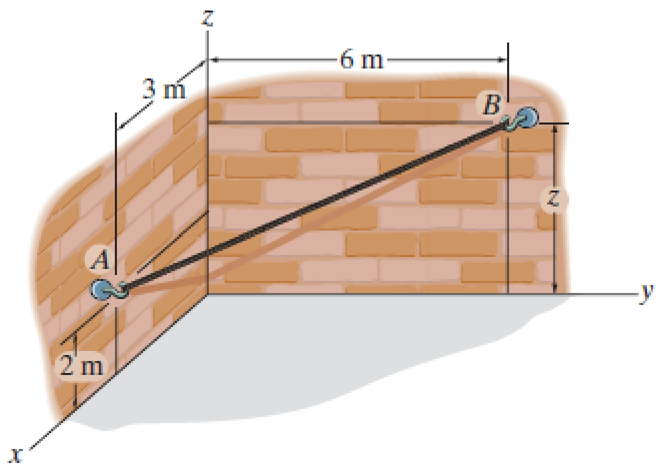
$F_2 = 180\text{ N}$ **Ans**

$\alpha_2 = 147°$ **Ans**

$\beta_2 = 119°$ **Ans**

$\gamma_2 = 75.0°$ **Ans**

2–87. If the cord *AB* is 7.5 m long, determine the coordinate position +*z* of point *B*.



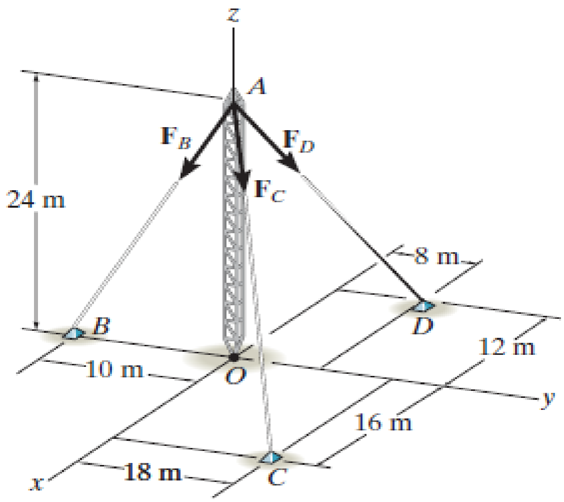
Position Vector: The coordinates for points *A* and *B* are $A(3, 0, 2)$ m and $B(0, 6, z)$ m, respectively. Thus,

$$\mathbf{r}_{AB} = (0-3)\mathbf{i} + (6-0)\mathbf{j} + (z-2)\mathbf{k}$$
$$= \{-3\mathbf{i} + 6\mathbf{j} + (z-2)\mathbf{k}\} \text{ m}$$

Since the length of cord is equal to the magnitude of $\mathbf{r}_{AB}$, then

$$r_{AB} = 7.5 = \sqrt{(-3)^2 + 6^2 + (z-2)^2}$$
$$56.25 = 45 + (z-2)^2$$
$$z - 2 = \pm 3.354$$
$$z = 5.35 \text{ m}$$ **Ans.**

2–104. The antenna tower is supported by three cables. If the forces of these cables acting on the antenna are $\mathbf{F_B}$ = 520 N, $\mathbf{F_C}$ = 680 N, and $\mathbf{F_D}$ = 560 N, determine the magnitude and coordinate direction angles of the resultant force acting at *A*.



$$\mathbf{F}_B = 520\left(\frac{\mathbf{r}_{AB}}{r_{AB}}\right) = 520\left(-\frac{10}{26}\mathbf{j} - \frac{24}{26}\mathbf{k}\right) = -200\,\mathbf{j} - 480\,\mathbf{k}$$

$$\mathbf{F}_C = 680\left(\frac{\mathbf{r}_{AC}}{r_{AC}}\right) = 680\left(\frac{16}{34}\mathbf{i} + \frac{18}{34}\mathbf{j} - \frac{24}{34}\mathbf{k}\right) = 320\,\mathbf{i} + 360\,\mathbf{j} - 480\,\mathbf{k}$$

$$\mathbf{F}_D = 560\left(\frac{\mathbf{r}_{AD}}{r_{AD}}\right) = 560\left(-\frac{12}{28}\mathbf{i} + \frac{8}{28}\mathbf{j} - \frac{24}{28}\mathbf{k}\right) = -240\,\mathbf{i} + 160\,\mathbf{j} - 480\,\mathbf{k}$$

$$\mathbf{F}_R = \Sigma\mathbf{F} = \{80\,\mathbf{i} + 320\,\mathbf{j} - 1440\,\mathbf{k}\}\ \text{N}$$

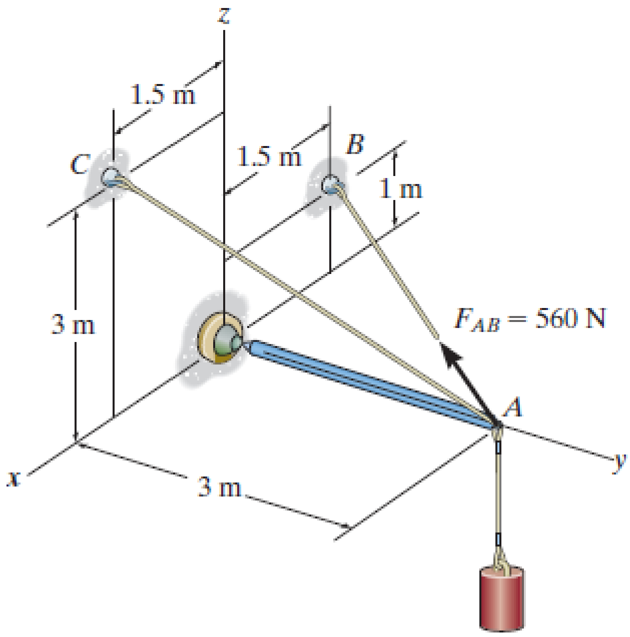
$$F_R = \sqrt{(80)^2 + (320)^2 + (-1440)^2} = 1477.3 = 1.48\ \text{kN} \qquad \textbf{Ans}$$

$$\alpha = \cos^{-1}\left(\frac{80}{1477.3}\right) = 86.9^\circ \qquad \textbf{Ans}$$

$$\beta = \cos^{-1}\left(\frac{320}{1477.3}\right) = 77.5^\circ \qquad \textbf{Ans}$$

$$\gamma = \cos^{-1}\left(\frac{-1440}{1477.3}\right) = 167^\circ \qquad \textbf{Ans}$$

2–112. Determine the projected component of the force. $\mathbf{F_{AB}}$ = 560 N acting along cable *AC*. Express the result as a Cartesian vector.



Force Vectors: The unit vectors $\mathbf{u}_{AB}$ and $\mathbf{u}_{AC}$ must be determined first. From Fig. *a*,

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{(-1.5-0)\mathbf{i}+(0-3)\mathbf{j}+(1-0)\mathbf{k}}{\sqrt{(-1.5-0)^2+(0-3)^2+(1-0)^2}} = -\frac{3}{7}\mathbf{i}-\frac{6}{7}\mathbf{j}+\frac{2}{7}\mathbf{k}$$

$$\mathbf{u}_{AC} = \frac{\mathbf{r}_{AC}}{r_{AC}} = \frac{(1.5-0)\mathbf{i}+(0-3)\mathbf{j}+(3-0)\mathbf{k}}{\sqrt{(1.5-0)^2+(0-3)^2+(3-0)^2}} = \frac{1}{3}\mathbf{i}-\frac{2}{3}\mathbf{j}+\frac{2}{3}\mathbf{k}$$

Thus, the force vector $\mathbf{F}_{AB}$ is given by

$$\mathbf{F}_{AB} = F_{AB}\,\mathbf{u}_{AB} = 560\left(-\frac{3}{7}\mathbf{i}-\frac{6}{7}\mathbf{j}+\frac{2}{7}\mathbf{k}\right) = [-240\mathbf{i}-480\mathbf{j}+160\mathbf{k}]\text{N}$$

Vector Dot Product: The magnitude of the projected component of $\mathbf{F}_{AB}$ is

$$(F_{AB})_{AC} = \mathbf{F}_{AB}\cdot\mathbf{u}_{AC} = (-240\mathbf{i}-480\mathbf{j}+160\mathbf{k})\cdot\left(\frac{1}{3}\mathbf{i}-\frac{2}{3}\mathbf{j}+\frac{2}{3}\mathbf{k}\right)$$

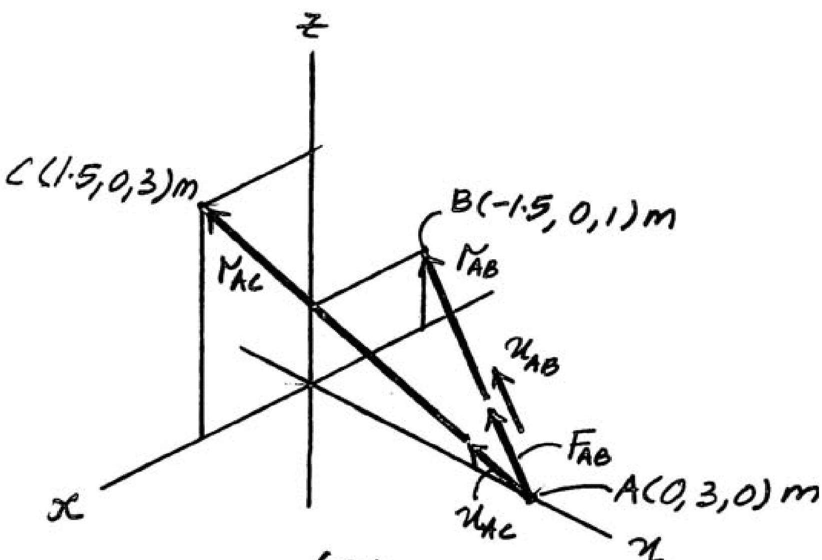
$$= (-240)\left(\frac{1}{3}\right)+(-480)\left(-\frac{2}{3}\right)+160\left(\frac{2}{3}\right)$$

$$= 346.67\text{ N}$$

Thus, $(\mathbf{F}_{AB})_{AC}$ expressed in Cartesian vector form is

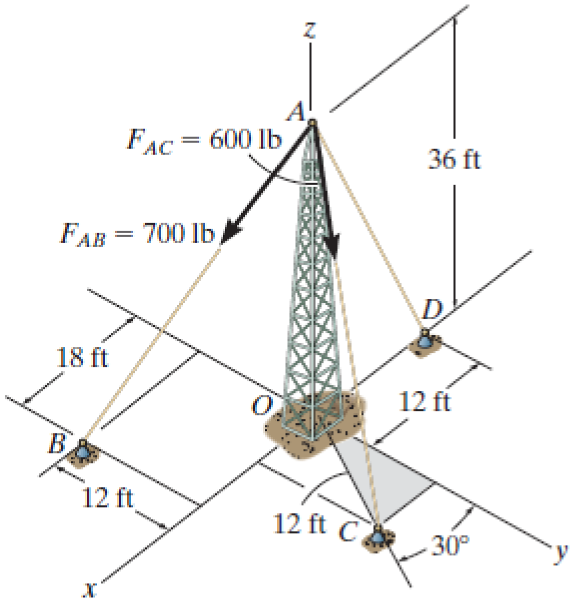
$$(\mathbf{F}_{AB})_{AC} = (F_{AB})_{AC}\,\mathbf{u}_{AC} = 346.67\left(\frac{1}{3}\mathbf{i}-\frac{2}{3}\mathbf{j}+\frac{2}{3}\mathbf{k}\right)$$

$$= [116\mathbf{i}-231\mathbf{j}+231\mathbf{k}]\text{N}$$ **Ans.**



(a)

2–121. Determine the magnitude of the projected component of force $\mathbf{F}_{AC}$ acting along the z axis.



Unit Vector: The unit vector $\mathbf{u}_{AC}$ must be determined first. From Fig. *a*,

$$\mathbf{u}_{AC} = \frac{\mathbf{r}_{AC}}{r_{AC}} = \frac{(12\sin 30° - 0)\mathbf{i} + (12\cos 30° - 0)\mathbf{j} + (0-36)\mathbf{k}}{\sqrt{(12\sin 30° - 0)^2 + (12\cos 30° - 0)^2 + (0-36)^2}} = 0.1581\mathbf{i} + 0.2739\mathbf{j} - 0.9487\mathbf{k}$$

Thus, the force vector $\mathbf{F}_{AC}$ is given by

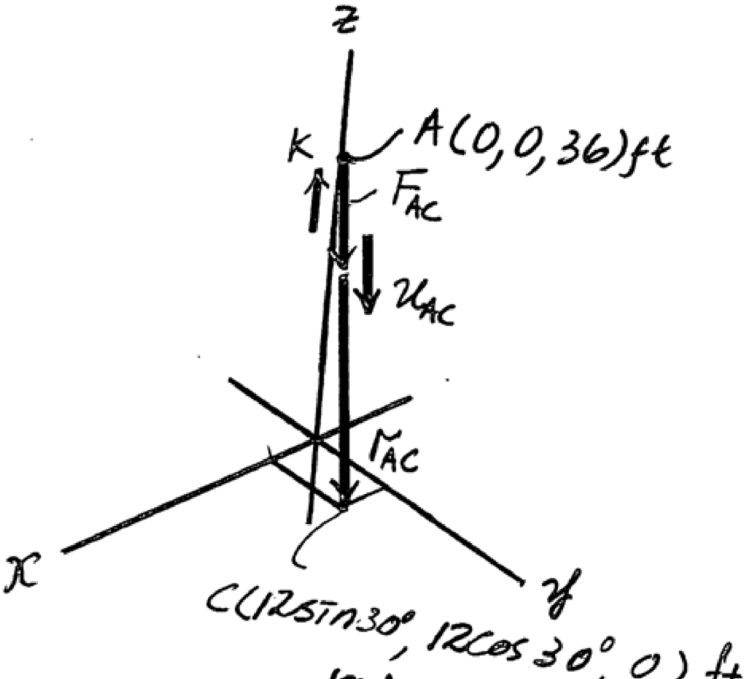
$$\mathbf{F}_{AC} = F_{AC}\mathbf{u}_{AC} = 600(0.1581\mathbf{i} + 0.2739\mathbf{j} - 0.9487\mathbf{k}) = \{94.87\mathbf{i} + 164.32\mathbf{j} - 569.21\mathbf{k}\}\text{N}$$

Vector Dot Product: The projected component of $\mathbf{F}_{AC}$ along the z axis is

$$(F_{AC})_z = \mathbf{F}_{AC} \cdot \mathbf{k} = (94.87\mathbf{i} + 164.32\mathbf{j} - 569.21\mathbf{k}) \cdot \mathbf{k}$$
$$= -569 \text{ lb}$$

The negative sign indicates that $(\mathbf{F}_{AC})_z$ is directed towards the negative z axis. Thus

$(F_{AC})_z = 569$ lb **Ans.**



(a)