1. (25)

a. (7) Using the electric field expression $E=\sigma/\varepsilon_0$ between two parallel plates with charge $+Q$ and $-Q$, area A and distance d between them, show that $C=\varepsilon_0 A/d$.

we Know that $C = \frac{Q}{\Delta V}$ by definition

$\Delta V = E.d$ $= \frac{\sigma}{\varepsilon_0} d$

$\sigma = \frac{Q}{A}$

$\Delta V_f = -\int \vec{E} \cdot d\vec{S}$

$C = \frac{Q}{\frac{\sigma}{\varepsilon_0}.d} = \frac{Q}{\frac{\frac{Q}{A}.d}{\varepsilon_0}} = \frac{\cancel{Q}}{\frac{\cancel{Q}.d}{A\varepsilon_0}} = \frac{\varepsilon_0 A}{d}$

5

b. (8) Two capacitors C_1 and C_2 are connected in parallel, demonstrate that the equivalence capacitor is $C=C_1+C_2$;

in parallel: ⇒ conservation of charge ⇒ $Q_{eq} = Q_1 + Q_2$

$C_{eq}\Delta V_{eq} = C_1 \Delta V_1 + C_2 \Delta V_2$

in parallel → conservation of energy ⇒ ΔV is the same

$\Delta V_{eq} = \Delta V_1 = \Delta V_2$

$C_{eq} \cancel{\Delta V} = \cancel{\Delta V}(C_1 + C_2)$

$\boxed{C_{eq} = C_1 + C_2}$

8

c. (10) Show that the energy stored in one capacitor per unit volume is independent of the its geometry (like area and distance)

$PE = \frac{1}{2} C(\Delta V)^2 \Rightarrow PE = \frac{1}{2}\left(\frac{\varepsilon_0 A}{d}\right)(E.d)^2$

$PE = \frac{1}{2} \frac{\varepsilon_0 A}{\cancel{d}} E^2 d^{\cancel{2}}$

$PE = \frac{1}{2} \varepsilon_0 A d E^2$

potential energy per unit volume:

$\frac{PE}{Vol} = \frac{\frac{1}{2}\varepsilon_0 A d E^2}{Vol}$ $\qquad u_E = \frac{\frac{1}{2}\varepsilon_0 \cancel{A}.\cancel{d} E^2}{\cancel{A}.\cancel{d}}$

$\boxed{u_E = \frac{1}{2}\varepsilon_0 E^2}$

does not depend on area A or distance d. between the plates.

10

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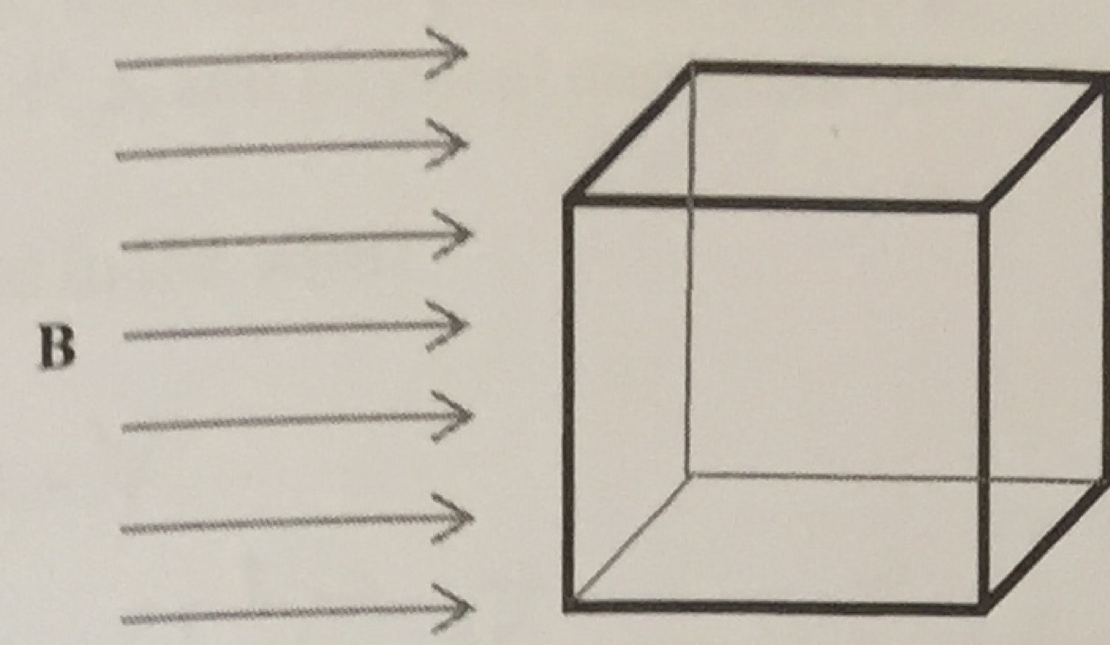
Score:________

☐ Check if solution is continued on the back.

II- **(20)** For a uniform magnetic field **B**, show in two different ways that the net magnetic flux through the cube is equal to 0.



1st way: By using Gauss's law, we Know that

$$\oint_{G.S} \vec{E} \cdot \vec{dA} = \text{electric flux}$$

20

By similarity $\oint_{G.S} \vec{B} \cdot \vec{dA} = \text{magnetic flux}.$

We choose the G.S. to be a cube centered at the center of the given cube and having the same dimesions and form.

$$\text{magnetic flux} = \oint_{G.S} \vec{B} \cdot \vec{dA} = \int_{\text{left side}} \vec{B} \cdot \vec{dA} + \int_{\text{rigth side}} \vec{B} \cdot \vec{dA} + \int_{\text{top}} \vec{B} \cdot \vec{dA} + \int_{\text{botton}} \vec{B} \cdot \vec{dA}$$
$$+ \int_{\text{back}} \vec{B} \cdot \vec{dA} + \int_{\text{front}} \vec{B} \cdot \vec{dA}$$

But in the back side, front side, top side and bottom side we have that the angle between $\vec{B}$ and $\vec{dA}$ is $90° \Rightarrow \vec{B} \perp \vec{dA} \Rightarrow \vec{B} \cdot \vec{dA} = B . dA \cos(90°) = 0$

$$\Rightarrow \text{magnetic flux} = \int_{\text{left side}} \vec{B} \cdot \vec{dA} + \int_{\text{right side}} \vec{B} \cdot \vec{dA} = \int_{\text{left}} B \times dA \times \cos(180°) + \int_{\text{right}} B \times dA \cos(0°)$$
$$= -B \int_{\text{left}} dA + B \int_{\text{right}} dA = -BA + BA = 0 \Rightarrow \boxed{\text{magnetic flux} = 0}$$

B is constant

2nd way: We Know that the magnetic flux correspond to the difference between the number of magnetic field lines getting into the cube and those getting out.

$\Rightarrow$ Since all the magnetic field lines going in the cube are the same going out of it

Therefore magnetic flux = field lines out − field lines in = 0

Score:______

☐ Check if solution is continued on the back.

III- **(30)** A straight cylindrical wire lying along the x axis has a length L and a diameter d. It is made of a material that obeys Ohm's law with a resistivity ρ. Assume that potential V is maintained at $x = 0$, and that the potential is zero at $x = L$. All answers should be in terms of L, d, V, ρ, and physical constants (no need to demonstrate Ohm's law).

a. **(5)** What is the expression for the electric field in the wire?

5

$\Delta V = E \cdot \ell$ ~~assuming~~ $\vec{E}$ is uniform

$E = \frac{\Delta V}{\ell} = \frac{V - 0}{L} = \frac{V}{L}$ $\boxed{E = \frac{V}{L}}$

b. **(5)** What is the resistance of the wire?

3

$R = \rho \frac{\ell}{A} = \cancel{\rho \frac{L}{L \cdot d}} = \cancel{\frac{\rho}{d}}$ $\boxed{\cancel{R = \frac{\rho}{d}}}$

$\Rightarrow \boxed{A = L \cdot d}$ $A = \frac{\pi d^2}{4}$

c. **(5)** What is the electric current in the wire?

2

Ohm's law: $\Delta V = RI$

$I = \frac{\Delta V}{R} = \cancel{\frac{E \ell}{\rho / d}} = \cancel{\frac{d E L}{\rho}}$ $\frac{\Delta V A}{\rho L}$ where $A = \frac{\pi d^2}{4}$ with $\sigma = \frac{1}{\rho}$

d. **(5)** Prove that $\mathbf{J} = \sigma \mathbf{E}$.

1

$j = \frac{I}{A} = \frac{\frac{dE\ell}{\rho}}{dL} = \cancel{\frac{dEL}{\rho d L}} = \cancel{\frac{E}{\rho}} = \cancel{\sigma E}$

e. **(10)** Show that $J = qnv_d$ and indicate what is n and v_d.

10

we find current I in terms of n, q, V_d, A,

$\Rightarrow$ $dQ = Nq$
$= (nV)q$
$= n(A \cdot dx)q$

$\Rightarrow$ $I = \frac{dQ}{dt} = \frac{(nq A \cdot dx)}{dt}$
$= nAqV_d$

~~we know~~ $V = \frac{dx}{dt} = x'$

$J = \frac{I}{A} = \frac{nAqV_d}{A} = nqV_d$

n: charge per unit volume ~~density~~

V_d is the drift velocity

Score:_______

☐ Check if solution is continued on the back.

e d] the charge in the length dL is:

$I = dQ/dt$ (2)

$$dQ = (n A\, dL)\, e \quad (1)$$

where n is the number of charge per unit volume.

$$\frac{dQ}{dt} = \left(n A \frac{dL}{dt}\right) e \quad (2)$$

$$\Rightarrow I = n A v_d e \quad \Rightarrow J = \frac{I}{A} = n v_d e \quad (2)$$

$$\Rightarrow \boxed{J = n v_d e} \quad (3) \quad \text{or} \quad \boxed{J = n v_d q}$$

v_d is the drift velocity.

d) $E = \frac{\Delta V}{L}$ (1)

$\Delta V = RI$ (1) (Ohm's law) (3)

$$\text{so } E = \frac{RI}{L} = \frac{RA}{L}\frac{I}{A}$$

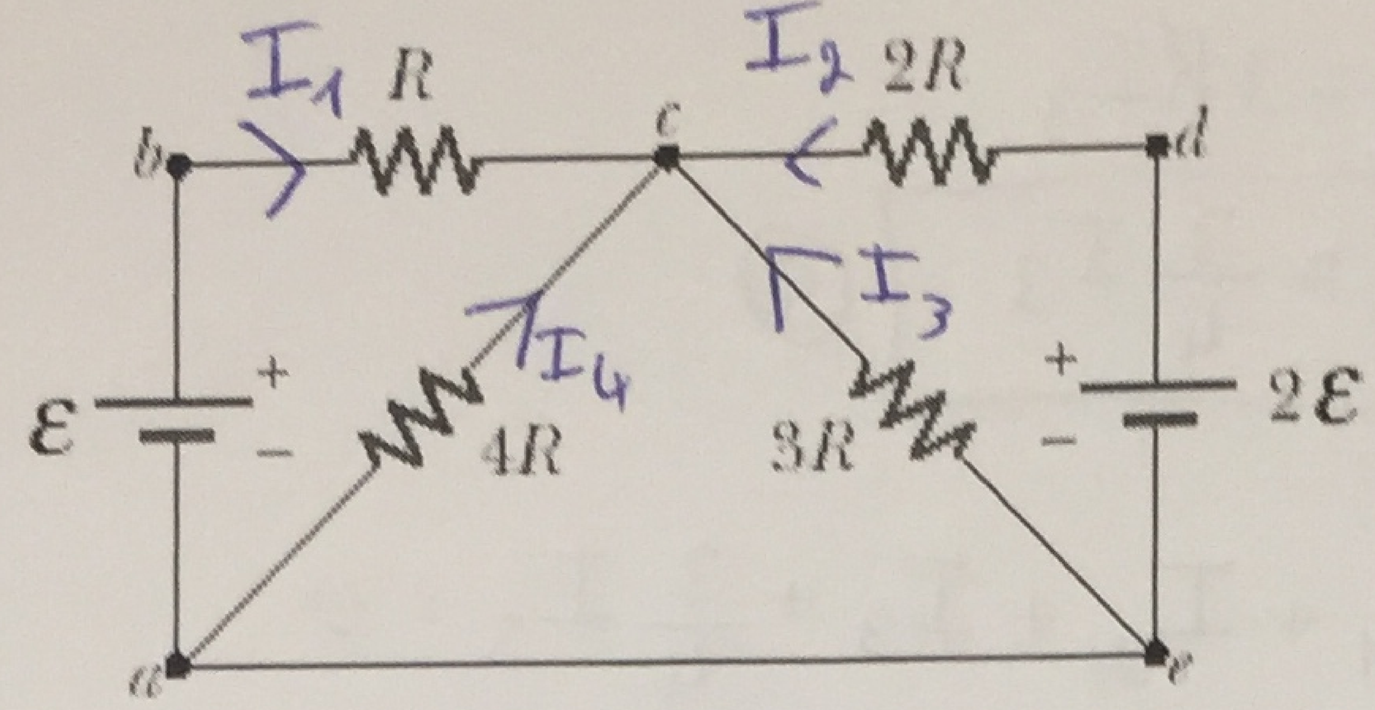
$$\Rightarrow E = \frac{RA}{L} J \Rightarrow \boxed{E = \rho J} \quad (6)$$

with $\rho = \frac{RA}{L}$

or J ... JE

25

IV- (25) As a function of $\mathcal{E}$ and R, determine the current in the right-hand side emf.



We choose the direction of the currents arbitrarely:

We apply Kirchhoff's junction rule at C: $\sum I_{\text{entering } C} = \sum I_{\text{exiting } C}$

$\Rightarrow \boxed{I_1 + I_2 + I_3 + I_4 = 0}$ ①

We apply Kirchhoff's loop rule for the loop: abca:

$\Delta V_{aa} = 0 \Rightarrow \Delta V_{ab} + \Delta V_{bc} + \Delta V_{ca} = 0$

$\mathcal{E} - RI_1 + 4RI_4 = 0$

$\Rightarrow \boxed{4RI_4 - RI_1 = -\mathcal{E}}$ ②

We apply Kirchhoff's loop rule for the loop: abdea:

$\Delta V_{aa} = 0 \Rightarrow \Delta V_{ab} + \Delta V_{bc} + \Delta V_{cd} + \Delta V_{de} + \Delta V_{ea} = 0$

$\mathcal{E} - RI_1 + 2RI_2 - 2\mathcal{E} + 0 = 0$

$\boxed{2RI_2 - RI_1 = \mathcal{E}}$ ③

We apply Kirchhoff's loop rule for the loop: ecde:

$\Delta V_{ee} = 0 \Rightarrow \Delta V_{ed} + \Delta V_{dc} + \Delta V_{ce} = 0$

$2\mathcal{E} - 2RI_2 + 3RI_3 = 0 \Rightarrow \boxed{+2RI_2 - 3RI_3 = 2\mathcal{E}}$ ④

Score:________

☑ Check if solution is continued on the back.

We apply Kirchhoff loop rule for the loop: ecae:

$\Delta V_{ee} = 0 \Rightarrow \Delta V_{ec} + \Delta V_{ca} + \Delta V_{ae} = 0$

$-3RI_3 + 4RI_4 = 0$

$\Rightarrow 4\cancel{R}I_4 = 3\cancel{R}I_3$

$\boxed{I_4 = \frac{3}{4}I_3}$ (5)

We replace in equation 1: $I_1 + I_2 + I_3 + \frac{3}{4}I_3 = 0$

$\boxed{I_1 + I_2 + \frac{7}{4}I_3 = 0}$

$\frac{(2)}{(3)} \Rightarrow \frac{\cancel{R}(4I_4 - I_1)}{\cancel{R}(2I_2 - I_1)} = \frac{-\mathcal{E}}{\mathcal{E}} = -1$

$\Rightarrow 4I_4 - I_1 = -2I_2 + I_1$

$\Rightarrow \boxed{3I_3 - 2I_1 + 2I_2 = 0}$

$\Rightarrow \begin{cases} 3I_3 - 2I_1 + 2I_2 = 0 \\ I_1 + I_2 + \frac{7}{4}I_3 = 0 \quad \times 2 \end{cases}$

$(+)\begin{cases} 3I_3 - 2I_1 + 2I_2 = 0 \\ 2I_1 + 2I_2 + \frac{7}{2}I_3 = 0 \end{cases} \Rightarrow 4I_2 + \frac{13}{2}I_3 = 0$

$\boxed{I_2 = -\frac{13}{8}I_3}$

Kirchhoff loop rule loop: edc:

$\Delta V_{ed} + \Delta V_{dc} + \Delta V_{ce} = 0$

$2\mathcal{E} - 2RI_2 + 3RI_3 = 0$

$2RI_2 = 2\mathcal{E} + 3RI_3$

$I_3 = \frac{-8}{13}I_2$

$I_2 = \frac{2\mathcal{E} + 3RI_3}{2R} = \frac{2\mathcal{E} + 3R\left(-\frac{8}{13}\right)I_2}{2R}$

~~$I_2 = 2\mathcal{E}$~~ -

$\Rightarrow 2RI_2 - 3R\left(-\frac{8}{13}\right)I_2 = 2\mathcal{E}$

$I_2 = \frac{2\mathcal{E}}{2R - 3R\left(-\frac{8}{13}\right)} \Rightarrow \boxed{I_2 = \frac{2\mathcal{E}}{2R + \frac{24R}{13}}}$

$\Rightarrow I_2 = \frac{2\mathcal{E}}{\frac{50R}{13}}$