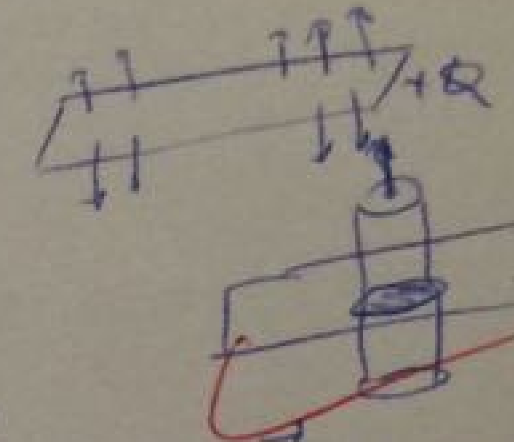
I. Consider an infinitely large and thin non-conducting sheet with charge
and area A.

a) (4) Show that the electric field **magnitude** at a distance x above the sheet is $E=\sigma/2\varepsilon_0$.



gauss's law: $\phi = \oint \vec{E}\cdot d\vec{A} = \frac{Q_{in}}{\varepsilon_0}$

gausian surface: cylinder with radius :

$\phi = \oint \vec{E}\cdot\vec{dA} = \int_{Top} \vec{E}\cdot\vec{dA} + \int_{Bot} \vec{E}\cdot\vec{dA} + \int_{around} \vec{E}\cdot\vec{dA}$ $\quad$ $\vec{E}$ is uniform.

$= E\int dA.\cos 0 + E\int dA \cos 0 + E\int dA \cos 90^\circ$

$= EA + EA$

$= 2EA$

here. $Q_{in} = \sigma A$

$2E\cancel{A} = \frac{\sigma A}{\varepsilon_0} \rightarrow \boxed{E = \frac{\sigma}{2\varepsilon_0}}$

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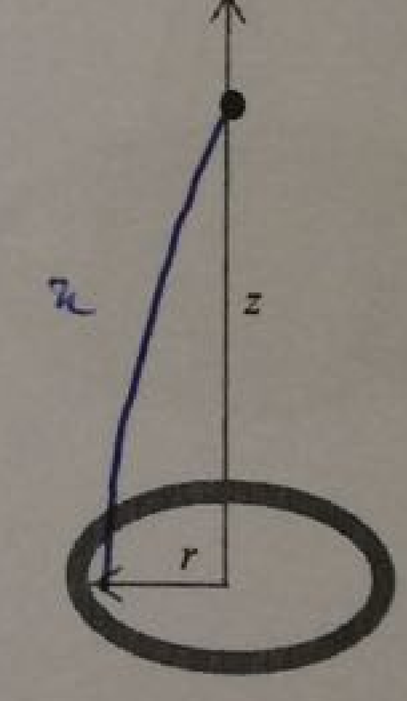
b) Consider now a uniformly charged ring (q_{ring}) with radius r and z *its* axis of symmetry. Show that the potential at a distance z from its center is $V_{ring} = k_e q_{ring}/\sqrt{r^2+z^2}$

we use $V = k_e \int \frac{dq}{x}$ $\quad$ considering $\int dq$ as the charge of the ring



here $x^2 = r^2 + z^2$ (phytagore)

$x = \sqrt{r^2+z^2}$

$V = k_e \int \frac{dq}{\sqrt{r^2+z^2}}$ $\quad$ $\sqrt{r^2+z^2}$ is constant

$V = \frac{k_e}{\sqrt{r^2+z^2}} \int dq \Rightarrow V = \frac{k_e Q_{ring}}{\sqrt{r^2+z^2}}$

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Score:________

☐ Check if solution is continued on the back.

c) A point charge $Q=q_{ring}$ of mass M is located initially at the center of the ring. When it is displaced slightly, the point charge accelerates along the z axis to infinity. Show that the ultimate speed of the point charge is

$$v = \left(\frac{2k_eQ^2}{MR}\right)^{1/2}$$

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By conservation of energy:

$(TE)_i = (TE)_f$

$KE_i + PE_i = KE_f + PE_f$

* $KE_i = \frac{1}{2} m V_i^2 = 0$ $\quad V_i = 0$

$PE_i = KE_f$

* $PE_f = q \cdot V_f = \frac{K_e q^2}{\sqrt{r^2+z^2}}$

$qV_i = \frac{1}{2} m V_f^2$

$\frac{K_e q^2}{\sqrt{r^2+0^2}} = \frac{1}{2} m V_f^2$

as $z \to \infty$
$PE_f \to 0$

$\frac{K_e Q^2}{r} = \frac{1}{2} m V_f^2$

$V_f^2 = \frac{2 K_e Q^2}{m r}$ $\quad \boxed{V = \sqrt{\frac{2K_e Q^2}{m r}}}$

d) Now, we wish to determine the potential V at a distance z of a **thin** disk with radius R and total charge $+Q$. Show that $V = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{R^2+z^2} - z\right)$

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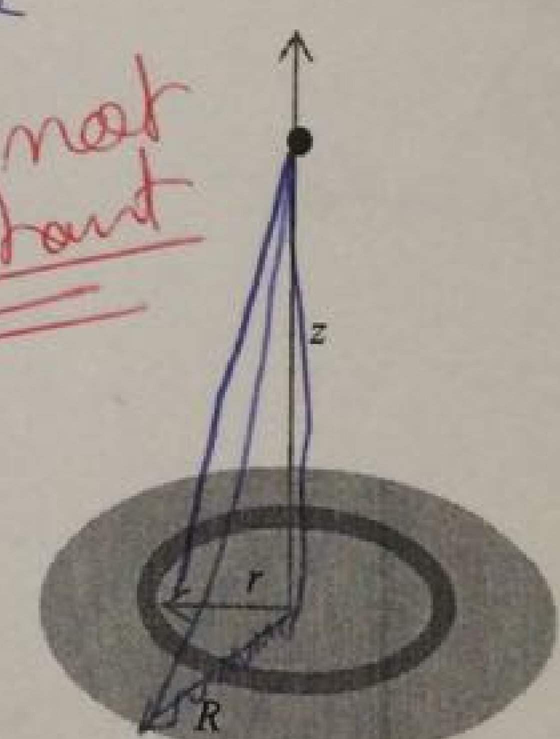
$V = K_e \int \frac{dq}{x}$ $\quad dq = \sigma \cdot dA$

$V = \frac{1}{4\pi\epsilon_0} \int \frac{\sigma \cdot dA}{x}$

x is a distance
So x = cte

x here is not constant



$V = \frac{\sigma}{(4\pi\epsilon_0)x} \int dA$

$= \frac{\sigma}{(4\pi\epsilon_0)x} 4\pi R^2$

$= \frac{\sigma R^2}{\epsilon_0 x}$

here the distance
x is $\sqrt{R^2+z^2}$

e) Obtain the THREE components of the electric field and discuss under what condition, it will be equal to that obtained using Gauss's law.

$$E_x = -\frac{dV}{dx} = 0$$

$$E_y = -\frac{dV}{dx} = 0$$

$$E_z = -\frac{dV}{dz} = -\frac{d\left(\frac{\sigma}{2\varepsilon_0}\sqrt{R^2+z^2} - z\right)}{dz}$$

$$= -\frac{\sigma}{2\varepsilon_0}\frac{d\left(\sqrt{R^2+z^2}-z\right)}{dz} = -\frac{\sigma}{2\varepsilon_0}\left(\frac{2z}{2\sqrt{R^2+z^2}} - 1\right)$$

$$\boxed{E_z = \frac{z\sigma}{2\varepsilon_0\sqrt{R^2+z^2}} - \frac{\sigma}{2\varepsilon_0}}$$

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f) What is the electric flux across a sphere centered at z with radius $a<z$?

$\cancel{E_z = \frac{z\sigma}{2\varepsilon_0\sqrt{R^2+z^2}}}$ ~~with the radius a~~ $\cancel{E_z = \frac{a\sigma}{2\varepsilon_0\sqrt{a^2+z^2}}}$

$$\boxed{\phi = \oint_{S} \vec{E}\cdot d\vec{A} = \frac{Q_{in}}{\varepsilon_0} \qquad \text{here } Q_{in} = 0 \quad \text{so } \phi = 0}$$

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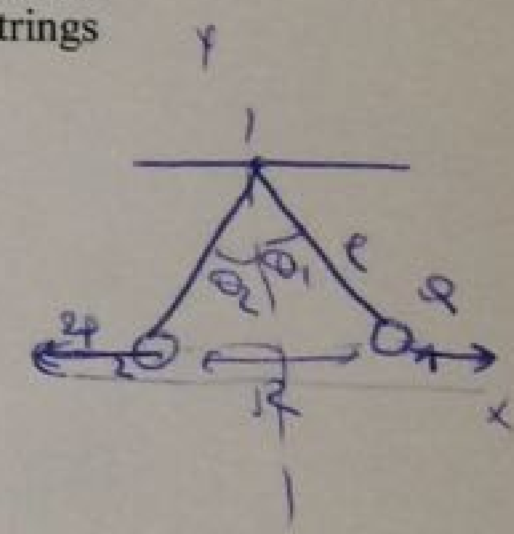
II. Two small spheres of mass m are suspended from strings of length ℓ that are connected at a common point. One sphere has charge Q; the other has charge $2Q$. At equilibrium, the strings make angles θ_1 and θ_2 with the vertical.

(a) How are θ_1 and θ_2 related?



$\boxed{\text{at equilibrium } F_{1/2} = -F_{2/1}}$

~~project on the y axis:~~

~~the angles θ_1 and θ_2 are equal~~

$\cancel{\theta_1 = \theta_2}$

$\cancel{\theta_2 = 2\theta_1}$

the charge of the first sphere is twice the charge of the second,

$\vec{F}_{1/2} = q\vec{E}_{(2)}$ $\quad \vec{F}_{2/1} = 2q\vec{E}_{(1)}$

so $\boxed{\theta_2 = 2\theta_1}$

☐ Check if solution is continued on the back.

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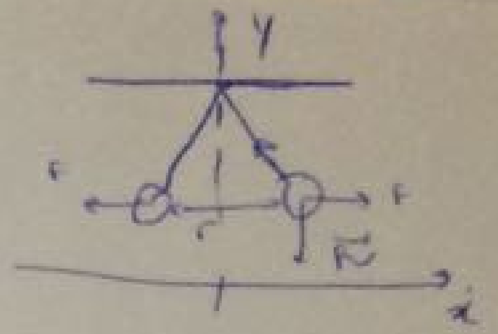

(b) Show that the distance r between the spheres is given by: $r \approx \left(\frac{4k_e Q^2 \ell}{mg}\right)^{1/3}$

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at epuilibrium $\sum \vec{F} = \vec{0}$ (we are studying epuilibrium at sphere 1)

$\vec{W} + \vec{T} + \vec{F}_{1/2} = \vec{0}$

this is for part a)

Project on the y axis:

$W_y + T_y + F_{y_1} = 0$ $\quad$ $F_{1/2\,y} = 0$ case $\perp$ to y axis

$-m.g + T\cos\theta_1 = 0$

Project on x axis

$W_x + T_x + F_{1/2\,x} = 0$ $\quad$ $W_x = 0$ cause $\perp$ to x axis

$-T\sin\theta_1 + F_{1/2\,x} = 0$

Coulomb's law: $F_{1/2\,x} = \frac{k_e (q)(2q)}{r^2} = \frac{k_e\, 2q^2}{r^2}$

(1) $\boxed{F_{1/2\,x} = T\sin\theta}$

(2) $\boxed{m.g = T\cos\theta_1}$

$\frac{(1)}{(2)} = \frac{F_{1/2\,x}}{m.g} = \tan\theta_1$

$F_{1/2\,x} = m.g\tan\theta_1$

$\frac{k_e\, 2q^2}{r^2} = m.g.\tan\theta_1$ $\quad$ $\tan\theta = \frac{r/2}{\ell}$

$r^2 = \frac{k_e\, 2q^2}{m.g.\tan\theta_1}$ $\quad$ $\frac{1}{\tan\theta} = 2\ell$

$r = \sqrt{\frac{k_e\, 2q^2}{m.g\tan\theta_1}}$

$r = \left(\frac{k_e\, 4q^2\ell}{m.g}\right)^{1/2}$

$R^2\sqrt{k^2 + 3^2} - 3$

☐ Check if solution is continued on the back.

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question 1,a : with charge q and area A.
question 1,d : see answer in Dr ghassan antar slides.
question 1,e,part2 : ask a friend idk :p
question 2,a: solved in part B

I'm sorry this previous is not a 100% neat.
good luck.