PHYSICS 211

Quiz II

Time: 70 minutes

Friday November 14th 2014

DO NOT OPEN THE EXAM BEFORE YOU ARE TOLD TO BEGIN
ONLY STANDARD CALCULATORS ARE ALLOWED
NO DOCUMENTS ALLOWED
YOU MAY DETACH THE SCRATCH PAPERS

NAME ______________________________________

ID Number ___________________________________

Section _____

Useful Information:

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \frac{d}{d\theta}\tan\theta = \sec^2\theta$$

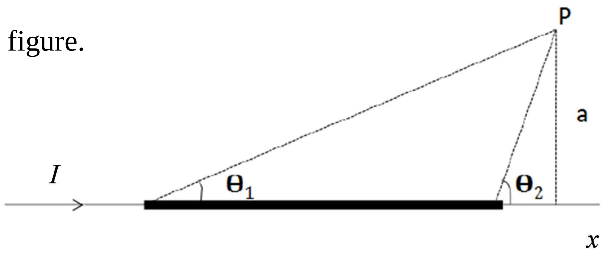
$$\sec\theta = \frac{1}{\cos\theta} \qquad \frac{d}{d\theta}\sec\theta = \sec\theta\tan\theta$$

$$\cot\theta = \frac{1}{\tan\theta} \qquad \frac{d}{d\theta}\cot\theta = -\csc^2\theta$$

$$\csc\theta = \frac{1}{\sin\theta} \qquad \frac{d}{d\theta}\csc\theta = -\csc\theta\cot\theta$$

I. **(25 points) ~20 minutes**

1. **(20)** Consider a thin, straight wire of finite length carrying a constant current I and placed along the x-axis as shown in the figure below. **a)** Give the direction of the magnetic field, **b)** then show that the magnetic field at point P is: $B = \frac{\mu_0 I}{4\pi a}(\cos\theta_1 - \cos\theta_2)$ where the angles θ_1 and θ_2 are as shown in the figure.



a) The magnetic field is out of the page (along $\hat{k}$) **2 points**

b)

$$d\vec{B} = \frac{\mu_0 I}{4\pi}\frac{d\vec{s}\times\hat{r}}{r^2}$$ **2 points**

$$d\vec{s}\times\hat{r} = |d\vec{s}\times\hat{r}|\hat{k} = dx\sin\theta\,\hat{k} \Rightarrow dB = \frac{\mu_0 I}{4\pi}\frac{dx\sin\theta}{r^2}$$ **2 points**

$$r = a/\sin\theta$$ **2 points**

$$x = -a/\tan\theta = -a\cot\theta$$ **3 points**

$$dx = -ad(\cot\theta) = a\csc^2(\theta)\, d\theta = \frac{a\, d\theta}{\sin^2\theta}$$ **3 points**

$$dB = \frac{\mu_0 I}{4\pi}\frac{a\, d\theta}{\sin^2\theta}\frac{\sin^2\theta}{a^2}\sin\theta = \frac{\mu_0 I}{4\pi a}\sin\theta d\theta$$ **2 points**

$$\Rightarrow B = \frac{\mu_0 I}{4\pi a}\int_{\theta_1}^{\theta_2}\sin\theta d\theta = \frac{\mu_0 I}{4\pi a}[-\cos\theta]_{\theta_1}^{\theta_2}$$ **2 points**

$$B = \frac{\mu_0 I}{4\pi a}(\cos\theta_1 - \cos\theta_2)$$ **2 point**

2. **(5)** If the wire becomes infinitely long, what would then be the magnitude of the magnetic field ?

$$\theta_1 \xrightarrow[L\to\infty]{} 0$$ **1 point**

$$\theta_2 \xrightarrow[L\to\infty]{} \pi$$ **1 point**

$$\Rightarrow B = \frac{\mu_0 I}{4\pi a}(1-(-1)) = \frac{\mu_0 I}{2\pi a}$$ **3 points**

II. **(20 points) ~ 15 minutes**

1. **(15)** Show that the current inside a cylindrical wire of radius r and length L takes the form $I = nqv_dA$; **Identify each of the variables.**

Let a segment of the conductor be of length $\Delta x = v_d \Delta t$, where v_d is the velocity of the charge carriers parallel to the axis of the cylinder, and Δt is the time interval required for the charge carriers in the segment to move through a displacement equal to the length of the segment.

3 points

The number of charge carriers in a segment of the wire is $nA\Delta x$ where n is the charge carrier density, and $A = \pi r^2$ is the cross-sectional surface of the wire.

3 points

During the time interval Δt all the charge carriers in the segment of length Δx would pass through the cross-sectional area at one end of the segment. Therefore the amount of charge that passes through this cross-sectional area during the time interval Δt is

3 points

$$\Delta Q = (nA\Delta x) q = (nAv_d \Delta t) q$$

3 points

Thus, we find that the current in the conductor is

$$I = \frac{\Delta Q}{\Delta t} = nqv_d A$$

3 points

2. **(5)** An electric heater is constructed by applying a potential difference of 220 V across a Nichrome wire that has a total resistance of 8.00 Ω. Find the current carried by the wire and the power rating of the heater.

$$I = \frac{\Delta V}{R} = \frac{220\ \text{V}}{8.00\ \Omega} = 27.5\ \text{A}$$

2.5 points

$$P = I^2 R = (27.5\ \text{A})^2 (8.00\ \Omega) = 6.05\times 10^3\ \text{W}$$

2.5 points

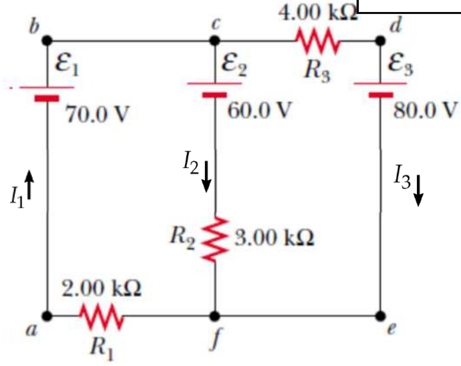
III. (25 points) ~15 minutes

1. **(5)** How many different currents are there in the circuit shown below? Choose arbitrary directions for these currents and show them on the figure.

2 points

There are 3 different currents in the circuit

3 points



2. **(15)** Find expressions giving the currents as functions of the *emf*s and the resistors.

3. **5)** Calculate the values of the unknown currents.

$I_1 = I_2 + I_3$ **3 points**

$\varepsilon_1 - \varepsilon_2 - V_{R_2} - V_{R_1} = 0 \Rightarrow \varepsilon_1 - \varepsilon_2 - I_2R_2 - I_1R_1 = 0$ **3 points**

$\varepsilon_2 - V_{R_3} - \varepsilon_3 + V_{R2} = 0 \Rightarrow \varepsilon_2 - I_3R_3 - \varepsilon_3 + I_2R_2 = 0$ **3 points**

$(1) \text{ in } (3) \Rightarrow I_3 = \dfrac{\varepsilon_2 - \varepsilon_3 + I_2R_2}{R_3}$ **1 points**

$\text{in } (2) \Rightarrow I_2 = \dfrac{(\varepsilon_1 - \varepsilon_2)R_3 - (\varepsilon_2 - \varepsilon_3)R_1}{R_2R_3 + R_1R_3 + R_1R_2} = 3.08 \text{ mA}$ **4 points**

$I_3 = -2.69 \text{ mA}$ **3 points**

$I_1 = 0.39 \text{ mA}$ **3 points**

IV. (30 points) ~20 minutes

1. **(5)** How is an ideal solenoid approached, and what are then the characteristics of its magnetic field?

An ideal solenoid is approached when the turns are closely spaced; **1.5 points** *and the length is much greater than the radius of the* **1.5 points**

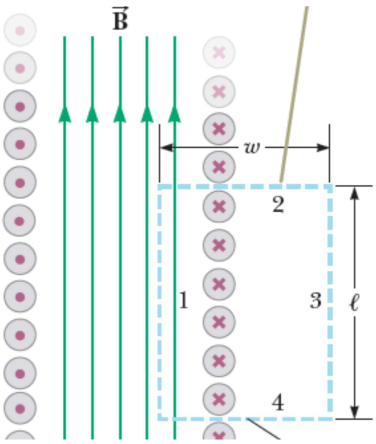
In this case, the external field is close to zero; **1 point** *and the interior field is uniform over a great volume.* **1 point**

2. **(10)** Use Ampère's law to find an expression for the magnitude of the magnetic field at the center of an ideal long solenoid carrying a current I as a function of the number of turns per unit length n. Explain your procedure.

We consider an Amperian loop with a rectangular path of length l and side w. **1 point**

We need to find $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}}$ *over the path of the loop to apply Ampère's law.* **1 point**

The contribution to this integral from side 2 and 4 are zero because the magnetic field is either perpendicular to the path (inside the solenoid) or negligible (outside the solenoid). **1 point**



The contribution from side 3 is also zero because we take the magnetic field outside the solenoid to be negligible. **1 point**

Furthermore, along side 1 the magnetic field is uniform and parallel to side 1, therefore : **1 point**

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \int_{\text{path 1}} \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \int_{\text{path 1}} B ds = B\int_{\text{path 1}} ds = Bl$$ **2 points**

If N is the number of turns in the length l of the amperian loop, the total steady current through the amperian loop is equal to NI; hence applying Ampère's law we get:

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = Bl = \mu_0 NI$$ **2 points**

$$B = \mu_0 \frac{N}{l} I$$

$$B = \mu_0 nI$$ **1 point**

where n is the number of turns per unit length.

3. **(15)** Now imagine we've inserted a straight current-carying conducting wire perpendicularly to the plane of the ideal solenoid, at its center. If this wire carries an upward current I_2 :

   a. **(5)** In which direction will the wire be deflected (left or right) ?

The wire will be deflected to the left. **5 points**

   b. **(10)** What is the magnitude of the magnetic force acting on the segment of the wire within the solenoid ? (Take R as the radius of the solenoid and use the magnitude of the magnetic field found in question IV.2.).

$\vec{F}_B = I_2 \vec{L} \times \vec{B}$ **3 points**

$\Rightarrow F_B = I_2 (2R) B \sin\frac{\pi}{2} = 2I_2 RB$ **3 points**

and $B = \mu_0 nI$

$\Rightarrow F_B = 2\mu_0 nRI_2 I$ **4 points**