PHYSICS 211
QUIZ 1
TIME: 60 minutes
October, 2014

DO NOT OPEN THIS EXAM BEFORE YOU ARE TOLD TO BEGIN

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ID Number ______________SOLUTION____________

Grading

A	
B	
TOTAL	

Useful information

$k_e = 8.99 \times 10^9 \ \text{N} \cdot \text{m}^2/\text{C}^2$

$\varepsilon_0 = 8.85 \times 10^{-12} \ \text{C}^2 / \text{N} \cdot \text{m}^2$

$e = 1.60 \times 10^{-19} \ \text{C}$

Part A: Multiple Choice Questions: (50%)

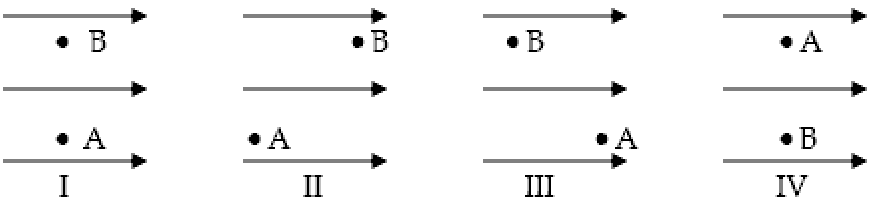
I. **(8%)** A point charge *Q* is placed at the origin. A second charge, 2*Q*, is placed on the *x* axis at *x* = -3.0 m. If *Q* = 50 μC, what is the magnitude of the electrostatic force on a third point charge, -*Q*, placed on the *y* axis at *y* = +4.0m?

a. 2.5 N
b. 3.7 N
c. 3.0 N
d. 4.4 N
e. 1.8 N

II. **(8%)** A charge of uniform linear density (4.0 nC/m) is distributed along the entire *x* axis. Determine the magnitude of the electric field on the *y* axis at *y* = 2.5 m.

a. 50 N/C
b. 36 N/C
c. 58 N/C
d. 29 N/C
e. 43 N/C

III. **(5%)** Four electrons move from point A to point B in a uniform electric field as shown below. Rank the electrons in diagrams I through IV by the changes in potential from most positive to most negative when traveling from A to B.



a. I = II = III = IV
b. II = III > I > IV
c. III > I = IV > II
d. II > I = IV > III
e. I > II = III > IV

IV. **(8%)** For the potential $V = 3x^3z - 2yz^2$, what is the corresponding electric field at the point (2,2,2)?

a. $-24\hat{\mathbf{i}} + 16\hat{\mathbf{j}} + 36\hat{\mathbf{k}}$
b. $-72\hat{\mathbf{i}} + 8\hat{\mathbf{j}} - 8\hat{\mathbf{k}}$
c. $24\hat{\mathbf{i}} - 16\hat{\mathbf{j}} - 36\hat{\mathbf{k}}$
d. $-72\hat{\mathbf{i}} + 16\hat{\mathbf{j}} - 36\hat{\mathbf{k}}$
e. The correct answer is not given

V. **(8%)** A 6.0 μF capacitor charged to 50 V and a 4.0 μF capacitor charged to 34 V are connected to each other, with the positive plate of each connected to the negative plate of the other. What is the total energy stored in the 6.0 μF capacitor at equilibrium?

a. 1.2 mJ
b. 5.7 mJ
c. 0.68 mJ
d. 0.97 mJ
e. 13 mJ

Score:________

☐ Check if solution is continued on the back.

VI. **(5%)** A parallel plate capacitor of capacitance C_0 has plates of area A with separation d between them. When it is connected to a battery of voltage V_0, it has a charge of magnitude Q_0 on its plates. It is then disconnected from the battery and the plates are pulled apart to a separation 2d without discharging them. After the plates are 2d apart, the magnitude of the charge on the plates and the potential difference between them are

a. $\frac{1}{2}Q_0, \frac{1}{2}V_0$

b. $Q_0, \frac{1}{2}V_0$

c. $2Q_0, 2V_0$

d. Q_0, V_0

e. $Q_0, 2V_0$

VII. **(8%)** A 16 nC charge is distributed uniformly along the x axis from $x = 0$ to $x = 4$ m. Which of the following integrals is correct for the magnitude (in N/C) of the electric field at $x = +10$ m on the x axis?

a. $\int_0^4 \frac{36dx}{x^2}$

b. $\int_6^{10} \frac{154dx}{(10-x^2)}$

c. $\int_0^4 \frac{36dx}{(10-x^2)}$

d. $\int_0^4 \frac{154dx}{x^2}$

e. None of these

Score:________

☐ Check if solution is continued on the back.

Part B: Problems: (50%)

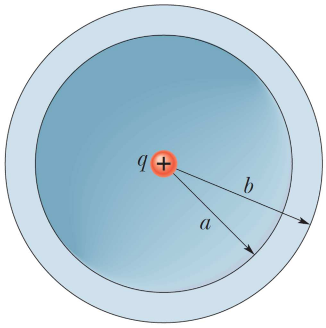
I. Gauss's law and Electric Potential (17%)

A nonconducting spherical shell of inner radius a and outer radius b has (within its thickness) a positive volume charge density $\rho = M / r$, where M is a constant and r is the distance from the center of the shell. In addition, a small ball of charge q is located at that center.

(a) **(9%)** Using Gauss's law, find the electric field in the shell ($a \le r \le b$).

(b) **(5%)** Find the potential difference between the inner radius a and the outer radius b of the spherical shell.

(c) **(3%)** What value should M have if the electric field in the shell is to be uniform?



a) Use a Gaussian surface in the form of a sphere of radius r_g concentric with the spherical shell and within it ($a < r_g < b$).

The charge that is both in the shell and within the Gaussian sphere is :

$$q_s = \int \rho dV = \int_a^{r_g} \rho 4\pi r^2 dr = 2\pi M \left(r_g^2 - a^2\right)$$

The total charge inside the Gaussian surface is

$$q_{in} = q + q_s = q + 2\pi M \left(r_g^2 - a^2\right)$$

Applying Gauss's law:

$$\Phi = \int \vec{\mathbf{E}} \cdot \mathbf{d}\vec{\mathbf{A}} = \frac{q_{in}}{\varepsilon_0}$$

$$4\pi r_g^2 E = \left(q + 2\pi M \left(r_g^2 - a^2\right)\right) / \varepsilon_0$$

We get

$$E = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{r_g^2} + 2\pi M - \frac{2\pi M a^2}{r_g^2}\right]$$

b)

$$\Delta V = -\int_a^b \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -\int_a^b E dr = -\frac{1}{4\pi\varepsilon_0}\left[-\frac{q}{r} + 2\pi M r + \frac{2\pi M a^2}{r}\right]_a^b$$

$$= -\frac{1}{4\pi\varepsilon_0}\left(-\frac{q}{b} + 2\pi M b + 2\pi M \frac{a^2}{b} + \frac{q}{a} - 4\pi M a\right)...$$

c) We must cancel the dependence on radius in E for E to be uniform:

$$\frac{q}{r^2} - \frac{2\pi M a^2}{r^2} = 0 \rightarrow M = \frac{q}{2\pi a^2}$$

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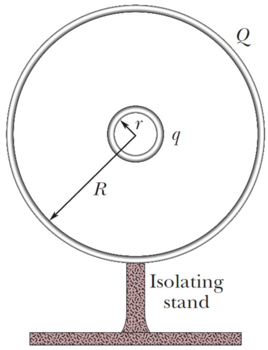
II. Electric Potential (13%)

A metal sphere with charge $q = 5.00\ \mu\text{C}$ and radius $r = 3.00\ \text{cm}$ is concentric with a larger metal sphere with charge $Q = 15.0\ \mu\text{C}$ and radius $R = 6.00\ \text{cm}$.

(a) **(5%)** What is the potential difference between the spheres? (Hint: *As for the electric field, the electric potential outside or on the external surface, of a spherically symmetric charge distribution is equivalent to that of a point charge*)

(b) **(8%)** If we connect the spheres with a wire, what then is the charge on the smaller sphere and the larger sphere?



a)

$$\Delta V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R} - \frac{1}{4\pi\varepsilon_0}\frac{q}{r} = 7.49\times 10^5\ \text{V}$$

b) By connecting two metal spheres with a wire, we now have one conductor, and any excess charge must reside on the surface of the conductor once electrostatic equilibrium is reached. Therefore, the charge on the small sphere is zero, and the charge on the large sphere is $Q' = Q + q = 20.0\ \mu\text{C}$.

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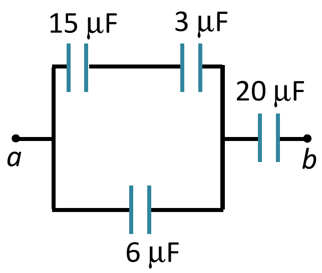
III. Capacitance (20%)

Four capacitors are connected as shown in the figure. The potential difference ΔV_{ab} is 15.0 V.

- **(a) (5%)** Find the equivalent capacitance between points *a* and *b*.
- **(b) (3%)** Calculate the charge on the 20 μF capacitor and the potential difference across it.
- **(c) (8%)** Find the potential difference across the 3 μF capacitor.
- **(d) (4%)** What is the total energy stored in this combination?



a) $C_{eq} = 5.96\ \mu F$

b) $q = C_{eq}\Delta V_{ab} = 89.5\ \mu C$

$\Delta V_{20} = 4.47\ V$

c) $\Delta V_3 = 8.77\ V$

d) $U = \frac{1}{2}C_{eq}(\Delta V_{ab})^2 = 6.7\times10^{-4}\ J$

Score:________

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SCRATCH PAPER

Nothing on this page will be graded

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