PHYSICS 211
Quiz I
TIME: 60 minutes

February 27, 2013

Key

DO NOT OPEN THIS EXAM BEFORE YOU ARE TOLD TO BEGIN

NAME____

ID Number ________________________

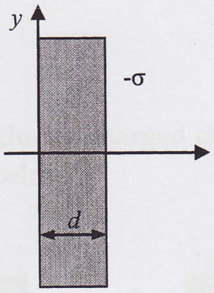
Grading

1 (35)	
2 (15)	
3 (15)	
4 (15)	
5 (20)	
TOTAL	

Score:________

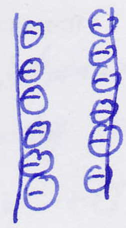
☐ Check if solution is continued on the back.

1- **(35)** Consider a sheet of a conducting metal charged negatively with a charge density $-\sigma$. The thickness of the sheet is d and its area is A with $A >> d$ in a way that the sheet can be considered as **infinitely** wide.



a) **(5)** The figure above illustrates a cut through this object. Illustrate and explain briefly how the charges are distributed in this sheet.

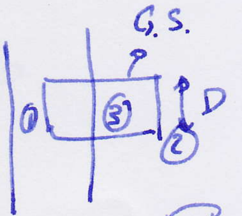
Since the sheet is in electrostatic equilibrium the charges live only on its surface

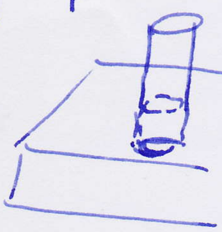


b) **(15)** Show that the electric field **magnitude** at a distance $x>d$ from the object surface is $E=\sigma/\varepsilon_0$.

- We use Gauss's law: $\phi_E = \int_{G.S.} \vec{E}\cdot d\vec{A} = \frac{Q_{in}}{\varepsilon_0}$

- The Gaussian surface (G.S.) is a cylinder with area A and length l





$\Rightarrow \int = \int_{1} + \int_{2} + \int_{3}$

$\int_{1} \vec{E}\cdot d\vec{A} = 0$ since E inside $= 0$

$\int_{3} \vec{E}\cdot d\vec{A} = 0$ since $\theta = 90°$ everywhere

$\int_{2} \vec{E}\cdot d\vec{A} = EA$ since $\theta = 0°$, $E = cte$ on A.

- $Q_{in} = \sigma A \Rightarrow EA = \sigma A/\varepsilon_0 \Rightarrow \boxed{E = \sigma/\varepsilon_0}$

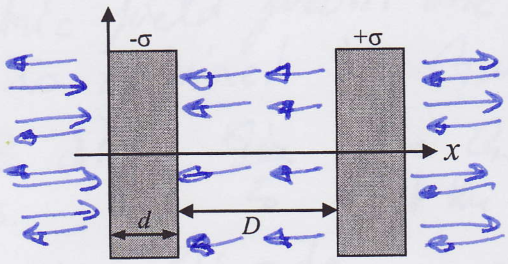
Score:_________

☐ Check if solution is continued on the back.

c) **(5)** What is the electric field direction and amplitude at a distance $0<x<d$?

$0<x<d$ means that we are inside the conductor where $\vec{E} = 0$.

d) Another identical conductor charged positively with $+\sigma$ is placed at a distance D as shown below.



- **(3)** Determine the electric field magnitude and direction at $x<0$

$E = \sigma/\varepsilon_0$ is independent of the distance to the plate. for $x<0$ the direction of E ~~is by~~ caused by $-\sigma$ is opposite to that caused by $+\sigma$

$$\Rightarrow E_T = E(-\sigma) - E(+\sigma) = 0$$

- **(3)** Determine the electric field at $d<x<D$

For $d<x<D$, the Two surfaces give an electric field in the same direction

$$\Rightarrow E_T = (E_{(-\sigma)} + E(+\sigma) = 2\sigma/\varepsilon_0$$

- **(4)** What is the potential different between the two plates?

We have the electric field $\Rightarrow$

$$\Delta V_{AB} = -\int \vec{E} \cdot d\vec{l} \quad \text{since } E = \frac{2\sigma}{\varepsilon_0} = \text{cte}$$

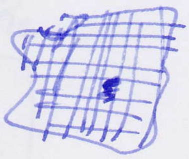
$$\Rightarrow \boxed{|\Delta V_{AB}| = \frac{2\sigma}{\varepsilon_0} D}$$

Score:________

☐ Check if solution is continued on the back.

2- **(15)** For an object charged with a total charge Q with an arbitrary shape we have seen that the electric field is given by $\vec{E} = k_e \int_{Object} \frac{dq}{r^2}\hat{r}$. Demonstrate how this equation was obtained.

We divide the arbitrary object into small point sources.



The electric field from one point source is obtained by Gauss's law

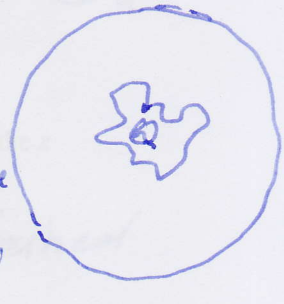
$\Phi_E = \int_{G.S.} \vec{E}\cdot d\vec{A} = \frac{Q_{in}}{\varepsilon_0}$; The G.S. is a sphere centered at the point source with radius r.

$\Rightarrow E(4\pi r^2) = \frac{Q_{in} = dq}{\varepsilon_0} \Rightarrow E = k_e \frac{dq}{r^2}$ where $k_e = \frac{1}{4\pi\varepsilon_0}$

Now integrating over all the point sources $\Rightarrow \vec{E} = k_e \int \frac{dq}{r^2}\hat{r}$.

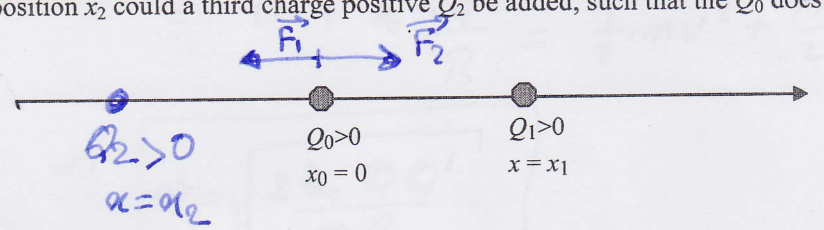
3- **(15)** What is the total electric flux across a sphere with radius r that is greater than an arbitrary charged object?

The electric flux of an arbitrary object across a sphere which fully contains the charge is obtained using Gauss's law



$\Phi_E = \frac{Q_{in}}{\varepsilon_0}$ here $Q_{in} = Q \Rightarrow \boxed{\Phi_E = \frac{Q}{\varepsilon_0}}$

4-**(15)** Consider the configuration shown below, with a positive charge Q_0 at position $x_0 = 0$ and another positive charge Q_1 at position x_1 along the x-axis.
At which position x_2 could a third charge positive Q_2 be added, such that the Q_0 does not move?



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In order for Q_0 not to move, Q_2 and since it is positive must be placed on the other side.

$$\Rightarrow \sum \vec{F} = 0 \Rightarrow \vec{F}_1 + \vec{F}_2 = 0 \Rightarrow \text{After projection on } \vec{x}$$

$$-F_1 + F_2 = 0 \Rightarrow F_1 = F_2$$

$$\Rightarrow \cancel{k_e} \frac{Q_1 \cancel{Q_0}}{x_1^2} = \cancel{k_e} \frac{Q_2 \cancel{Q_0}}{x_2^2} \Rightarrow x_2^2 = \frac{Q_1}{Q_2} x_1^2$$

$$\Rightarrow \boxed{x_2 = \pm \sqrt{\frac{Q_1}{Q_2}}\, x_1} \Rightarrow \boxed{x_2 = -\sqrt{\frac{Q_1}{Q_2}}\, x_2}$$

5- (20) The x axis is the symmetry axis of a stationary uniformly charged ring of radius R and charge Q. A point charge Q' of mass M is located initially at the center of the ring.

a) Determine the potential at the center of the ring

We use $dV = k_e \frac{dQ}{R} \Rightarrow V = k_e \int \frac{dQ}{R}$

since $R = C^{te}$ for all dQ's $\Rightarrow$

$$V = k_e \frac{1}{R} \int dQ \Rightarrow \boxed{V = k_e \frac{Q}{R}}$$

b) When the charge Q' is displaced slightly, the point charge accelerates along the x axis to infinity. Determine the ultimate speed of the point charge.

The ultimate (final) speed is obtained using the conservation of energy:

$$(TE)_i = KE_i + PE_i = (TE)_f = KE_f + PE_f$$

$$= 0 + k_e \frac{QQ'}{R} = \frac{1}{2} m v^2 + 0$$

$\hookrightarrow x \to \infty \Rightarrow V \to 0$

$$\Rightarrow \boxed{v = \sqrt{\frac{2 k_e Q Q'}{m R}}}$$

Score:________

☐ Check if solution is continued on the back.