PHYSICS 211
Final EXAM, Fall 2011-2012
TIME: 120 minutes

January 23, 2012

DO NOT OPEN THIS EXAM BEFORE YOU ARE TOLD TO BEGIN

The usage of programmable calculators is strictly forbidden

NAME____

ID Number ____________________

Useful information
$\varepsilon_0 = 8.85\text{x}10^{-12}$ C^2/N.m^2
$\mu_0 = 4\pi$ x 10^{-7} Tm/A
$q = 1.6 \times 10^{-19}$ C
$m_e = 9.1\text{x}10^{-31}$ kg
$m_p = 1.67\times10^{-27}$ kg
$k_e = 8.9875\text{x}10^9$ Nm^2/C^2

Grading

Part I (20)	
Part II (20)	
Part III (30)	
Part IV (30)	
Total	

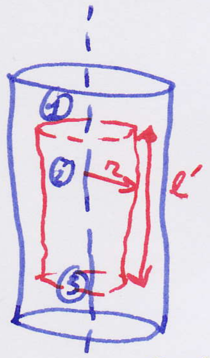
Score:________

☐ Check if solution is continued on the back.

Part I: Gauss's Law (20)

Consider a long cylinder with radius R and a length l that is very large when compared to R so as to neglect the cylinder's end edge effects. This long cylinder has a volumetric charge density ρ.

(a) (7) Show that the electric field **inside** the cylinder at a distance r from its axis has the expression $E = (\rho/2\varepsilon_0)r$.



To determine the electric field, we use Gauss's law

$\Phi_E = \int_{G.S.} \vec{E}\cdot d\vec{A} = \frac{Q_{in}}{\varepsilon_0}$; The Gaussian surface (G.S.) is a cylinder with radius r.

$\Phi_E = \Phi_1 + \Phi_2 + \Phi_3$ or $\Phi_1 = \Phi_3 = 0$ because $\vec{E} \perp \vec{A}$

$\Phi_2 = E(2\pi r l') = \frac{Q_{in}}{\varepsilon_0}$; or $\rho = \frac{Q}{V} = \frac{Q_{in}}{V_{G.S.}} \Rightarrow Q_{in} = Q\cdot\frac{\pi r^2 l'}{\pi R^2 l}$

$\Rightarrow \boxed{E = \frac{\rho}{2\varepsilon_0} r}$ $\qquad Q_{in} = \rho\pi r^2 l$

(b) (8) Show that the electric field **outside** the cylinder at a distance r from its axis has the expression $E = 2\pi k_e\rho R^2/r$

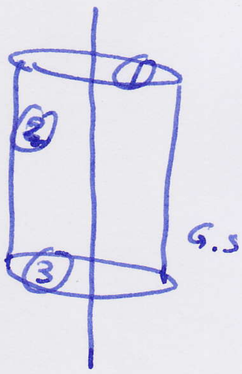
The Gaussian surface is now with $r > R$, it is a cylinder

$\Phi_E = \Phi_1 + \Phi_2 + \Phi_3 = E(2\pi r l') = \frac{\rho}{\varepsilon_0}\pi R^2 l'$

as $\Phi_1 = \Phi_3 = 0$

$\Rightarrow E = \frac{\pi}{2\pi\varepsilon_0}\rho\frac{R^2}{r} = 2k_e\rho\frac{R^2}{r}\pi$

(c) (5) When $R \to 0$ the wire can be described by a linear charge density **λ, show that** $\lambda = Q/l$.[Hint: Compare the electric field above to the one obtained with a linear charge density λ]



For a thin wire, the Gaussian surface is a cylinder with radius r and length l'

$\Phi_E = \Phi_1 + \Phi_2 + \Phi_3$; $\Phi_1 = \Phi_3 = 0$ as $\vec{E} \perp \vec{A}$

$\Rightarrow \Phi_2 = E(2\pi r l) = \frac{Q_{in}}{\varepsilon_0} \Rightarrow E = 2k_e\frac{Q_{in}}{r}$

$E = 2k_e\frac{Q_{in}}{r} = 2\pi k_e\rho\frac{R^2}{r} \Rightarrow Q_{in} = \pi R^2\rho = \frac{\pi R^2}{\pi R^2 l}Q = \frac{Q}{l}$

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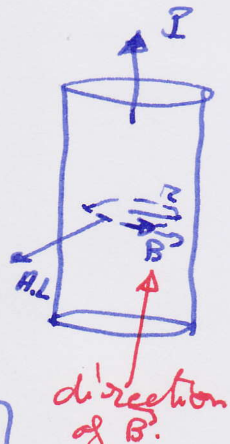
Part II: Ampere's Law (20)

(a) (4) Assume that the charges of the cylinder in part I are now moving with a drift velocity v_d, determine the expression of the current I.

$I = \frac{dQ}{dt}$ or $Q = Nq$ with $N = nV = nAx$

$\Rightarrow I = qnA\frac{dx}{dt}$ on the average $\boxed{I = nqAv_d}$

(b) (6) Show that the magnetic field **inside** the wire has the expression $B = \left(\frac{\mu_0 I}{2\pi R^2}\right)r$; make your own drawing to show its direction.



We use Ampère's law: $\oint_{A.L.} \vec{B}\cdot d\vec{l} = \mu_0 I_{in}$

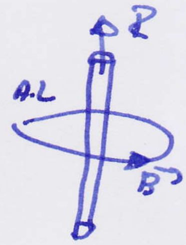
We choose the Amperian loop (A.L.) to be a circle with radius r.

$\Rightarrow \oint \vec{B}\cdot dl = \oint B\,dl = B\oint dl = B(2\pi r) = \mu_0 I_{in}$

The current is uniform $\Rightarrow$

$j = \frac{I}{A} = \frac{I_{in}}{A_{A.L.}} \Rightarrow I_{in} = \frac{\pi r^2}{\pi R^2} I \Rightarrow \boxed{B = \frac{\mu_0 I}{2\pi R^2}\cdot r}$

(c) (5) Show that the magnetic field **outside** the wire has the expression $B = \frac{\mu_0 I}{2\pi r}$; make a drawing to show its direction. (Same as above)



The Ampère's law: $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{in}$

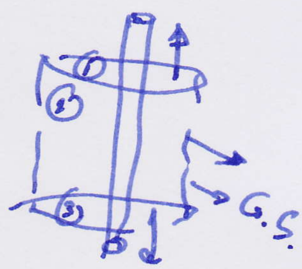
$\Rightarrow B(2\pi r) = \mu_0 I$ as $I_{in} = I$

$\Rightarrow \boxed{B = \frac{\mu_0 I}{2\pi r}}$

(d) (5) State and verify Gauss's law for magnetism using the wire with current I.

Gauss's law for magnetism reads: $\Phi_B = \int_{G.S.} \vec{B}\cdot d\vec{A} = 0$

We choose the ~~th~~ Gaussian surface to be a closed cylinder around the wire



$\Rightarrow \Phi_B = \Phi_1 + \Phi_2 + \Phi_3$

$\Phi_1 = \Phi_2 = \Phi_3 = 0$ as $\vec{B} \perp d\vec{A}$ everywhere

$\Rightarrow \boxed{\Phi_B = 0}$

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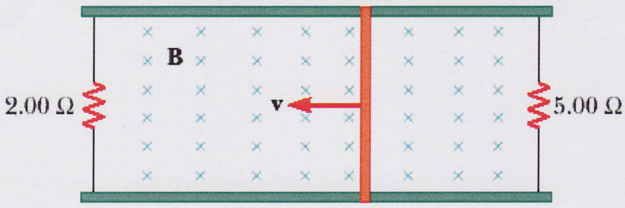
A conducting rod of length $\ell = 35.0$ cm is free to slide on two parallel conducting bars as shown in the Figure below. Two resistors $R_1 = 2.00\ \Omega$ and $R_2 = 5.00\ \Omega$ are connected across the ends of the bars to form a loop. A constant magnetic field $B = 2.50$ T is directed perpendicularly into the page. An external agent pulls the rod to the left with a constant speed of $v = 8.00$ m/s.



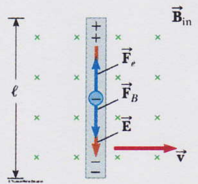
(a) 5 Using Faraday's law, find the **analytical expression** and the **value** of the emf caused by the motion of the bar.

Faraday's law states $\varepsilon = -\dfrac{d\Phi_B}{dt}$ where FB is the magnetic flux $\Phi_B = \int \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}}$
Take **A** to be a rectangle of length l and width dx and taking advantage of te fact that the magnetic field is constant leads to: $\varepsilon = -\dfrac{d\Phi_B}{dt} = -B\ell\dfrac{dx}{dt} = -B\ell v$

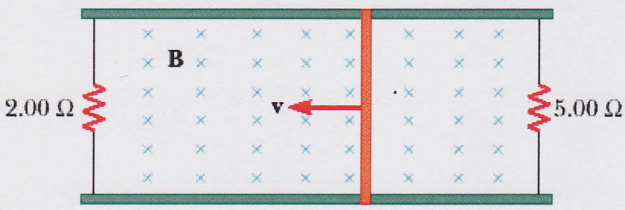
$\varepsilon = B\ell v = (2.50\ \mathrm{T})(0.350\ \mathrm{m})(8.00\ \mathrm{m/s}) = 7.00\ \mathrm{V}$

(b) 6 Consider the motion of free electrons in the bar. Show that the **same expression** of the emf can be found when equilibrium is reached between the electric and the magnetic forces.

Two forces are acting on free electrons, the magnetic force $\vec{F} = q\vec{v} \times \vec{B}$ and the electric force. At equilibrium, we have the sum of the forces is equal to 0 leading to
$qE = qvB$ or $E = vB$
The electric field is related to the potential difference across the ends of the conductor:
$\Delta V = E\ell = B\,\ell\,v$



(c) 4 Show on the figure above the direction of the currents in the two resistors, and justify below.



The left-hand loop contains decreasing flux away from you, so the induced current in it will be clockwise, to produce its own field directed away from you. Let I_1 represent the current flowing

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upward through the 2.00-Ω resistor. The right-hand loop will carry counterclockwise current. Let I_3 be the upward current in the 5.00-Ω resistor.

(d) 4 Determine the value of the current in both resistors.

Kirchhoff's loop rule then gives:

$$+7.00\text{ V} - I_1(2.00\ \Omega) = 0$$
$$I_1 = \boxed{3.50\text{ A}}$$

and $+7.00\text{ V} - I_3(5.00\ \Omega) = 0$ $I_3 = \boxed{1.40\text{ A}}$.

(e) 5 Find the total power delivered to the resistance of the circuit.

The total power dissipated in the resistors of the circuit is

$$P = \varepsilon I_1 + \varepsilon I_3 = \varepsilon(I_1 + I_3) = (7.00\text{ V})(3.50\text{ A} + 1.40\text{ A}) = \boxed{34.3\text{ W}}$$

(f) 6 Find the magnitude of the applied force that is needed to move the rod with this constant velocity.

Method 1: The current in the sliding conductor is downward with value $I_2 = 3.50\text{ A} + 1.40\text{ A} = 4.90\text{ A}$. The magnetic field exerts a force of $F_m = I\ell B = (4.90\text{ A})(0.350\text{ m})(2.50\text{ T}) = 4.29\text{ N}$ directed ☞ toward the right on this conductor. An outside agent must then exert a force of $\boxed{4.29\text{ N}}$ to the left to keep the bar moving.

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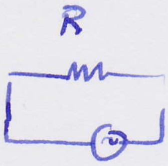
Part IV: AC circuits (30)

We consider a circuit composed of an AC power source, supplying a current $I = I_{\max}\sin(\omega t)$, a resistor R and an inductor L.

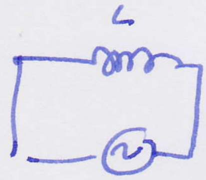
(a) (4) Consider that the **resistance alone** is connected to the power supply, determine the expression of the potential difference across it.



We use Ohm's law $\Delta v = R I$

$\Rightarrow \Delta v(t) = R I_{max} \sin \omega t \Rightarrow \begin{cases} \Delta V_{max} = R I_{max} \\ \varphi = 0 \end{cases}$

(b)(4) Consider that the **inductor alone** is connected to the power supply, determine the expression of the potential difference across it.

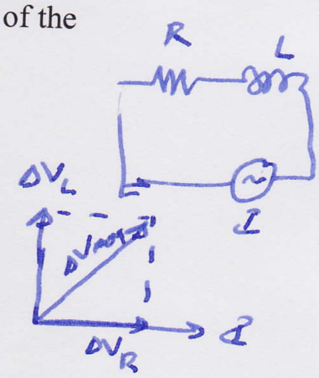


Kirchhoff's loop rule leads to

$\Delta V_L - L\frac{dI}{dt} = 0 \Rightarrow \Delta V = \omega L I_{max} \cos\omega t$

$\Rightarrow \Delta V = L\omega I_{max} \sin(\omega t + \pi/2) \Rightarrow \begin{cases} \Delta V_{L,max} = L\omega I_{max} \\ \varphi = +\pi/2 \end{cases}$

(c) (6) When both R and L are put in series, determine the impedance of the circuit.



We use the phasor diagram for

$\Delta V = \Delta V_L + \Delta V_R$

or $\Delta V_M = \sqrt{\Delta V_{L,max}^2 + \Delta V_{R,max}^2}$

$\Delta V_M = \sqrt{R^2 + L^2\omega^2} \times I_{max}$.

$\Rightarrow \boxed{Z = \sqrt{R^2 + L^2\omega^2}}$

(d)(5) Determine the expression of the circuit phase.

$\tan\varphi = \frac{\Delta V_{L,max}}{\Delta V_{R,max}} = \frac{L\omega I_{max}}{R I_{max}} \Rightarrow \boxed{\varphi = \tan^{-1}\left(\frac{L\omega}{R}\right)}$

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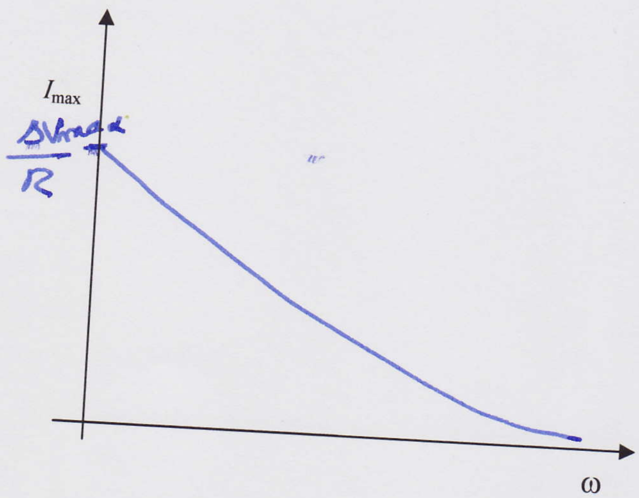
(e) (6) Use the axes below to plot the current I_{max} vs. ω assuming ΔV_{max} is constant and discuss the behavior of this circuit at low frequencies (ω→0) and at high frequencies (ω→∞).

as $\omega \to 0 \Rightarrow Z \to R \Rightarrow I_{max} \to \frac{\Delta V_{max}}{R}$

as $\omega \to \infty \Rightarrow Z \to \infty \Rightarrow I_{max} \to 0$

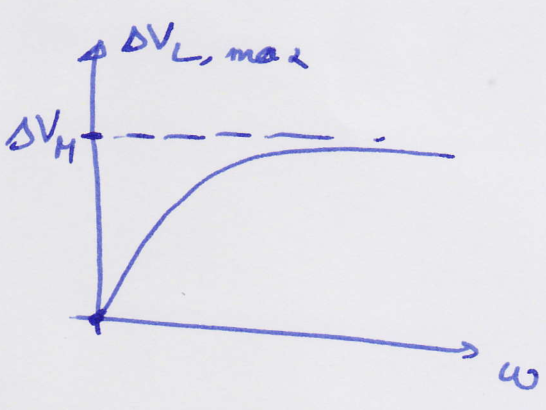


Find the relationship between the potential drop across the inductor and the power supply. What might one use this circuit for?

(f) (5) ~~Determine the voltage across the inductor and plot it as a function of the angular frequency.~~

$\Delta V_L = Z_L I \Rightarrow \Delta V_{L,max} = L\omega I_{max}$

$= \frac{L\omega}{\sqrt{R^2 + L^2\omega^2}} \Delta V_M$



This circuit can be used as a high-pass filter when oscillations at low frequency are not 'visible' around the inductor.