PHYSICS 211
Quiz II
TIME: 60 minutes

December 15, 2011

DO NOT OPEN THIS EXAM BEFORE YOU ARE TOLD TO BEGIN

NAME____

ID Number ____________________

Useful information
ε_0=8.85x10^{-12} C^2/N.m^2.

Grading

1	
2	
3	
4	
TOTAL	

Score:_______

☐ Check if solution is continued on the back.

1. **(18%)** Consider a battery producing a current I flowing in a long cylindrical wire with resistivity ρ, length l and diameter D. **[All the answers below should be given as a function of these parameters]**

a- **(6%)** Start from the definition of a current to show that $I=nqv_dA$ and use this calculation to define, n, q, v_d and A.

$I = \frac{dQ}{dt}$; $Q = Nq$ where q is the elementary charge.

$N = nV$ where $\begin{cases} n \text{ is the charge density} \\ V \text{ is the volume } = A.x \end{cases}$

$\Rightarrow Q = nqAx \Rightarrow I = nqA\frac{dx}{dt} = nqAv_d$

where v_d is the drift velocity

b- **(6%)** Does the potential drop across the resistor depends on A, justify.

In order to show ΔV dependence on A, we use Ohm's law: $\Delta V = RI$ or $R = \rho\frac{l}{A}$ and $I = nqv_dA$

$\Rightarrow \Delta V = \rho l n q v_d$ and does NOT depend on A.

c- **(6%)** What is the energy per unit volume dissipated in the wire in 1 second?

The dissipated power in the resistor is

$P = RI^2 = \frac{dW}{dt} \Rightarrow W = RI^2t$

$t = 1s \Rightarrow W = \rho\frac{l}{A}n^2q^2v_d^2A^2 \Rightarrow$

$\boxed{W = \rho l n^2 q^2 v_d^2 A}$

Score:________

☐ Check if solution is continued on the back.

2. (32%)

a- (8%) Using Ampere's law, show that the magnetic field generated at a distance *a* from the wire has the expression $B=\mu_0 I/2\pi a$.

Ampere's law $\oint_{A.L.} \vec{B}\cdot d\vec{s} = \mu_0 I_{in}$

The Amperian loop (A.L.) is a circle centered at the wire with radius a.

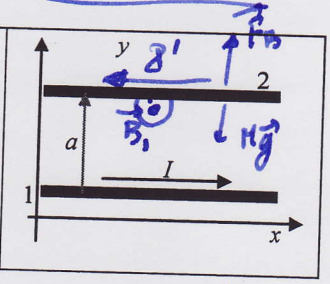
Everywhere on the loop we have $\vec{B}\cdot d\vec{s} = B\,ds$

$\Rightarrow \oint_{A.L} \vec{B}\,d\vec{s} = \oint_{A.L} B\,ds = B\,(2\pi a)$ since B is constant

at any a $\Rightarrow B(2\pi a) = \mu_0 I \Rightarrow$

$$\boxed{B = \frac{\mu_0 I}{2\pi a}}$$

b- (8%) Another wire (2) with mass *M* with a current *I'* is laid horizontally in free space at a distance *a* from the wire 1 which is laying also horizontally on the ground as shown in the figure to the right. Determine the current *I'* amplitude and direction necessary to keep the wire 2 from falling and in equilibrium.



Wire 2 is in equilibrium $\Rightarrow \sum \vec{F} = 0$

$\sum \vec{F} = \vec{W} + \vec{F_B}$ where $\vec{W} = M\vec{g} = Mg(-\hat{y})$

$\vec{F_B} = B_1 I' l\,\hat{y}$,

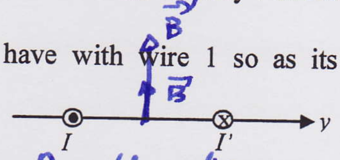
$\vec{F_B}$ needs to be in the $+\hat{y}$-direction $\Rightarrow I'$ is in the $-\hat{a}$-direction

$B_1 = \frac{\mu_0 I}{2\pi a} \Rightarrow$ After projection on the y-axis

we have $Mg = B_1 I' l \Rightarrow \boxed{I' = \frac{Mg}{B_1 l}} = \frac{Mg\,(2\pi a)}{\mu_0 I\, l}$

c- An electron at a distance y from wire 1 is moving at a constant velocity v in the x-direction.

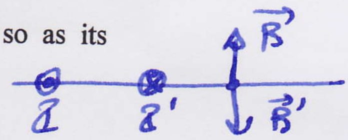
a. **(6%)** Determine the distance it has to have with wire 1 so as its trajectory is not modified when $y<a$



If the e^- is moving in the $+\hat{x}$-direction $\Rightarrow$ $\vec{B}$ and $\vec{B}'$ caused by I and I' are both directed to the top $\Rightarrow \Sigma\vec{F} \neq 0 \Rightarrow$

$\boxed{\text{No equilibrium distance.}}$

b. **(5%)** Determine the distance it has to have with wire 1 so as its trajectory is not modified when $y>a$.



In this cas $\vec{B}$ and $\vec{B}'$ have opposite directions $\Rightarrow \vec{F}_B$ and $\vec{F}_B'$ have opposite directions as well. Equilibrium if $\Sigma\vec{F} = 0$

$\Rightarrow F_B = F_B' \Rightarrow qvB = qvB' \Rightarrow \frac{\mu_0 I}{2\pi(a+y)} = \frac{\mu_0 I'}{2\pi y}$

$\Rightarrow \boxed{y = aI'/(I-I')}$

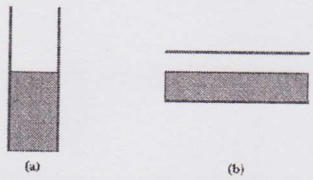
c. **(5%)** Where the electron can be placed to stay in equilibrium if it was not moving at all, justify?

If $\vec{v} = 0 \Rightarrow$ the e^- can be place anywhere since the magnetic force $= 0$.

☐ Check if solution is continued on the back.

Score:________

3. **(20%)** A vertical parallel-plate capacitor with area A and distance d between the plates, is **half** filled with a dielectric for which the dielectric constant is κ (figure (a) below).



(a)(6%) Determine the equivalent capacitance of the system when the two plates are positioned vertically as in (a).

Since the potential difference is the same on the filled and empty capacitor, they can be considered to be two capacitors in parallel

$$\Rightarrow C_{eq_1} = C_1 + C_2 = \frac{\varepsilon_0 (A/2)}{d} + \kappa \frac{\varepsilon_0 A/2}{d} = \left(\frac{\kappa+1}{2}\right) \frac{\varepsilon_0 A}{d}.$$

(b)(6%) Determine the equivalent capacitance of the system when the two plates are positioned horizontally and filled up to a height $x=fd$ where f is the fraction of the filling.

These two capacitors can be considered to be in series $\Rightarrow$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{\kappa\varepsilon \frac{A}{x}} + \frac{1}{\varepsilon \frac{A}{d-x}} ; \quad C_{eq_2} = \left[\frac{\kappa}{f + \kappa(1-f)}\right] \frac{\varepsilon_0 A}{d}.$$

(c)(8%) When this capacitor is positioned horizontally, determine the fraction f it should be filled with the same dielectric (κ) in order for the two capacitors to have equal capacitance?

$$C_{eq_1} = C_{eq_2} \Rightarrow \left(\frac{\kappa+1}{2}\right) \frac{\varepsilon_0 A}{d} = \left[\frac{\kappa}{f + \kappa(1-f)}\right] \frac{\varepsilon_0 A}{d}.$$

$$(\kappa+1)(f + \kappa(1-f)) = 2\kappa$$

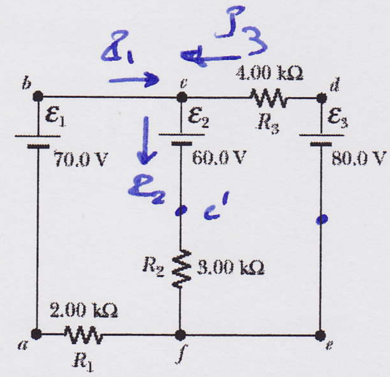
$$\Rightarrow f(1-\kappa) = \frac{2\kappa}{1+\kappa} - \kappa = \frac{\kappa - \kappa^2}{(1+\kappa)} = \frac{\kappa(1-\kappa)}{1+\kappa}$$

$$\Rightarrow \boxed{f = \frac{\kappa}{1+\kappa}}$$

Score:________

☐ Check if solution is continued on the back.

4. (30%) Kirchhoff's rules



(a) (4%) State the two rules of Kirchhoff and what each of them represents.

Kirchhoff's rules are:

The loop rule ⇒ $(\sum \Delta V)_{Loop} = 0$; it reflects conservation of energy

The junction rule ⇒ $(\sum I)_{IN} = (\sum I)_{out}$; it reflects conservation of charges

(b) (13%) find the current in each resistor in the Figure above.

We choose the currents arbitrarily

⇒ The junction rule ⇒ $\boxed{I_1 + I_3 = I_2}$

The 1st loop rule : $\Delta V_{ab} + \Delta V_{bc} + \Delta V_{cc'} + \Delta V_{c'f} + \Delta V_{fa} = 0$

$\Rightarrow +\mathcal{E}_1 + 0 - \mathcal{E}_2 - R_2 I_2 - R_1 I_1 = 0$

$\Rightarrow \boxed{R_2 I_2 + R_1 I_1 = \mathcal{E}_1 - \mathcal{E}_2}$

The 2nd loop :

$\Delta V_{ab} + \Delta V_{bd} + \Delta V_{da} = 0$

$\mathcal{E}_1 + R_3 I_3 - R_1 I_1 = 0 \Rightarrow \boxed{R_3 I_3 - R_1 I_1 = \mathcal{E}_1}$

$$\Rightarrow \begin{cases} I_1 = 0.385 \text{ mA} \\ I_3 = 2.69 \text{ mA} \\ I_2 = 3.08 \text{ mA} \end{cases}$$

Score:________

☐ Check if solution is continued on the back.

(b)(5%) Find the potential difference between points c and f. Which point is at the higher potential?

$$\Delta V_{fc} = V_c - V_f = \Delta V_{cc'} + \Delta V_{c'f}$$
$$= -\,\mathcal{E}_2 - I_2 R_2 = -69.2\,V$$

$\Rightarrow V_f$ is at higher potential

(c)(3%) Find the potential between points e and f

Since no element is in between e and f

$\Rightarrow \Delta V_{ef} = 0$

(e) (5%) Compare the power dissipated to the power generated and comment.

The Power dissipated by the resistors is

$P_1 = R_1 I_1^2 = \cancel{286.15\,W} = 0.296\ mW.$

$P_2 = R_2 I_2^2 = 28.46\ mW$

$P_3 = 29\ mW.$

Concerning the batteries when the current is in the same direction $\Rightarrow$ It will act as a generator when it is the opposite it will act as a dissipator

$\Rightarrow$ $P_{\mathcal{E}_1} = +I_1 \Delta V_1 = +I_1 \mathcal{E}_1 = 26.95$

$P_{\mathcal{E}_3} = +I_3 \Delta V_3 = +I_3 \mathcal{E}_3 = 215.2$

$P_{\mathcal{E}_2} = +I_2 \Delta V_2 = -I_2 \mathcal{E}_2 = 184.8.$

$\Rightarrow$ Generated Power : $P_T = 242\ W$

Dissipated " : $P_T = 242\ W$

They are equal