**PHYSICS 211**
**Quiz I**
**TIME: 60 minutes**

October 25, 2011

**DO NOT OPEN THIS EXAM BEFORE YOU ARE TOLD TO BEGIN**

NAME________________________________________

ID Number ________________________

Section ________________________

SOLUTION

Key

Useful information
$\varepsilon_0 = 8.85 \times 10^{-12}$ $C^2/N.m^2$.

**Grading**

| | |
|---|---|
| A | |
| B | |
| | |
| TOTAL | |



Score:

☐ Check if solution is continued on the back.

## Part A: Multiple choice questions (18%)

1. (3%) Two charged particles, $Q_1$ and $Q_2$, are a distance $r$ apart with $Q_2 = 5Q_1$. Compare the forces they exert on one another when $\mathbf{F}_1$ is the force $Q_2$ exerts on $Q_1$ and $\mathbf{F}_2$ is the force $Q_1$ exerts on $Q_2$.

   a. $F_2 = 5F_1$.
   b. $F_2 = -5F_1$.
   c. $F_2 = F_1$.
   d. $F_2 = -F_1$.
   e. $5F_2 = F_1$.

2. (3%) A balloon is charged with $Q$ and inflated leading to the increase of its radius, how does the electric field inside and outside (at a distance $r$ from the center) changes?

   a. Increase and increase
   b. Increase and decrease
   c. Decrease and decrease
   d. Decrease and increase
   e. None of the above

3. (3%) A hemispherical surface (half of a spherical surface) of radius $R$ is located in a uniform electric field of magnitude $E$ that is parallel to the axis of the hemisphere. What is the magnitude of the electric flux through the hemisphere surface?

   a. $\pi R^2 E$
   b. $4\pi R^2 E/3$
   c. $2\pi R^2 E/3$
   d. $\pi R^2 E/2$
   e. $\pi R^2 E/3$



Score:________

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4. (3%) A spaceship encounters a single plane of charged particles, with the charge per unit area equal to $\sigma$. The electric field a short distance above the plane has magnitude _____ and is directed _____ to the plane.

a. $\frac{\sigma}{2\varepsilon_0}$, parallel

b. $\frac{\sigma}{2\varepsilon_0}$, perpendicular

c. $\frac{\sigma}{\varepsilon_0}$, parallel

d. $\frac{\sigma}{\varepsilon_0}$, perpendicular

e. $\frac{2\sigma}{\varepsilon_0}$, parallel

5. (3%) Equipotentials are lines along which

a. the electric field is constant in magnitude and direction.
b. the electric charge is constant in magnitude and direction.
c. maximum work against electrical forces is required to move a charge at constant speed.
d. a charge may be moved at constant speed without work against electrical forces.
e. charges move by themselves.

6 (3%) A series of 3 uncharged concentric shells surround a small central charge $q$. The charge distributed on the outside of the third shell is

a. $-3q$.
b. $-(\ln 3)q$.
c. $+q$.
d. $+(\ln 3)q$.
e. $+3q$.



Score:________

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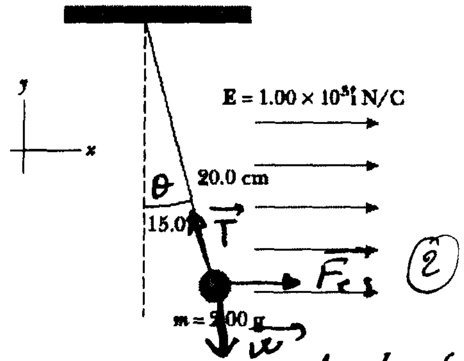
## Part B- Problems (82 %)

**1. (20%)** A small, 2.00-g plastic ball is suspended by a 20.0-cm-long string in a uniform electric field as shown in the Figure below. If the ball is in equilibrium when the string makes a 15.0° angle with the vertical, what is the net charge on the ball?




At equilibrium, we use Newton's law : $\sum \vec{F} = 0$

(5) $\sum \vec{F} = \vec{W} + \vec{T} + \vec{F}_{es} = 0$. where $\vec{F}_{es} = q\vec{E}$ (4)

We project this equation on x : $qE - T\sin\theta = 0$

on y : $-W + T\cos\theta = 0$ (4)

$\Rightarrow q = \frac{T\sin\theta}{E}$ or $T = \frac{W}{\cos\theta} \Rightarrow \boxed{q = \frac{W\,\mathrm{tg}\,\theta}{E}}$

$\boxed{q = 5.25\times10^{-6}\ C}$ (5)



Score:________

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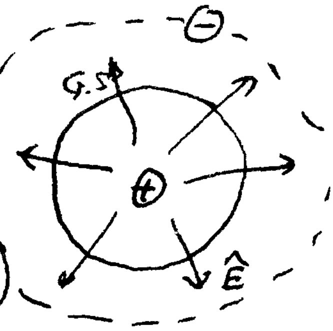
**2.(27%) Gauss law**

**a.(11%)** Consider a hydrogen atom formed of one proton and one electron assumed to circulate around at a radius $a$. Using Gauss's law, determine the electric field at a radius $r<a$.

- Gauss's law state $\Phi_E = \int_{G.S} \vec{E}\, d\vec{A} = \frac{Q_{in}}{\varepsilon_0}$ (2)
- We select the Gaussian surface (G.S.) to be a sphere centered on the proton. (2)




$$\Rightarrow \Phi_E = \underbrace{\int_{G.S} E \cdot dA}_{\cos\theta = 1 \text{ everywhere}} = \underbrace{E \int_{G.S.} dA}_{E \text{ is the same at } r.} = EA = E(4\pi r^2) \quad (4)$$

- $Q_{in} = +q$ (2) $\Rightarrow \boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} = k_e \frac{q}{r^2}}$ (1)

**b.(8%)** What is the total flux across a cube centered at the proton and with side $x=a/3$?

The total flux across a cube centered on the proton $\Phi_{cube} = \frac{Q_{in}}{\varepsilon_0}$ according to Gauss's law since the G.S. is arbitrary (3) $\Rightarrow \boxed{\Phi_{cube} = \frac{q}{\varepsilon_0}}$ (5)

**c. (8%)** What is the electric field at $r>a$?

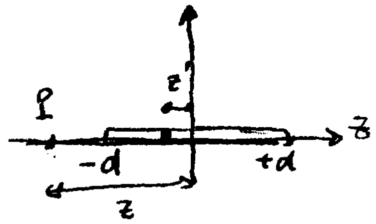
At $r>a$, Gauss's law yields $\Phi_E = \int_{G.S} \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\varepsilon_0}$

or $Q_{in}(\text{for } r>a) = 0 = +q - q$ (3) $\Rightarrow$

$\boxed{E = 0}$ (5)

**3.(35%)** A thin rod extends along the $z$-axis from $z = -d$ to $z = d$. The rod carries a positive charge $Q$ uniformly distributed along its length $2d$ with charge density $\lambda = Q/2d$.

**(a)(12%)** Calculate the electric potential at a point $z > d$ along the $z$-axis.




$\lambda = \frac{Q}{2d}$. let $dz$ denote an element charge $dq \Rightarrow dq = \lambda dz$ (2)

(4)

$$dV = k_e \frac{dq}{z - z'} = k_e \lambda \frac{dz'}{z - z'}$$

$$\Rightarrow V(z) = \int dV = k_e \lambda \int_{-d}^{+d} \frac{dz'}{z - z'} = \boxed{k_e \lambda \log\left(\frac{z+d}{z-d}\right)}$$

(2) (4)

**(b)(8%)** What is the change in potential energy if an electron moves from $z = 4d$ to $z = 3d$?

$$V(z = 4d) = k_e \lambda \log\left(\frac{4d + d}{4d - d}\right) = k_e \lambda \log\left(\frac{5}{3}\right) \quad (2)$$

$$V(z = 3d) = k_e \lambda \log 2 \quad (2)$$

$$\Rightarrow \Delta V = \underbrace{V(z = 3d)}_{final} - \underbrace{V(z = 4d)}_{initial} = k_e \lambda \log\left(\frac{6}{5}\right) \quad (2)$$

The Potential energy $\Delta U = q \Delta V$

$$= \boxed{-e k_e \lambda \log\left(\frac{6}{5}\right)} \quad (2)$$



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**(c)(7%)** If the electron started out at rest at the point $z = 4d$, what is its velocity at $z = 3d$?

We use the conservation of energy

$$\Rightarrow \Delta KE = -\Delta U \quad 4$$

$$\frac{1}{2}mv_f^2 - 0 = +e k_e \lambda \log\left(\frac{6}{5}\right)$$

$$\Rightarrow v_f = \sqrt{\frac{2|e|\lambda k_e}{m}\log\left(\frac{6}{5}\right)} \quad 3$$

**(d)(8%)** What is the electric field at a point $z > d$?

The electric field is obtained from the potential by differentiation

$$\Rightarrow 2\; E_x = -\frac{\partial V}{\partial x} = 0$$

$$2\; E_y = -\frac{\partial V}{\partial y} = 0$$

V does not depend on x and y

$$E_z = -\frac{\partial V}{\partial z} = -k_e\lambda \frac{\partial}{\partial z}\left[\log\left(\frac{z+d}{z-d}\right)\right] = \frac{2k\lambda d}{z^2-d^2} = \boxed{\frac{KQ}{z^2-d^2}}$$

4



Score:________

☐ Check if solution is continued on the back.