Name: $\qquad$
Section number: $\qquad$
Instructor's Name: $\qquad$
ID number: $\qquad$ DO NOT START THE EXAM BEFORE YOU ARE TOLD TO BEGIN

Grading

| I |  |
| :---: | :--- |
| II-1 |  |
| II-2 |  |
| II-3 |  |
| TOTAL |  |

The duration of this exam is 60 minutes.
No notes or books allowed.
Scientific calculators are allowed
All results should be given with the exact number of significant figures.

## I. (50\%)

In the "Standing Waves on a Stretched String" experiment, the string length is changed and the corresponding resonant frequencies are measured. The hanging mass is $\mathrm{M}=12 \mathrm{Kg} \pm 6 \%$

The following results were obtained for the $\mathbf{2}^{\text {nd }}$ Harmonic standing waves.

| $\mathrm{L}(\mathrm{cm})$ | $\mathrm{f}(\mathrm{KHz})$ |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 50 | 1.297 |  |  |  |  |
| 60 | 0.901 |  |  |  |  |
| 70 | 0.662 |  |  |  |  |
| 80 | 0.507 |  |  |  |  |
| 90 | 0.400 |  |  |  |  |
|  |  |  |  |  |  |

You may find the formulae on page 4 useful.
a- Write down the necessary equation. (10\%)
b- Choose your variables such that you obtain a linear relationship between them. (10\%)
c- Use linear regression to find the slope of your line along with its error. (15\%)
d- Determine from the slope the linear density of the string along with its error. (15\%)

## Linear Regression

The method of least squares is used to fit a curve (find a theoretical equation) to a set of experimental data. First assume that a linear relation exists between $y$ and $x$

$$
\begin{equation*}
y=A x+B \tag{1}
\end{equation*}
$$

Substitution of $x=x_{i}$ will in general not give the value of $y_{i}$. The "errors" will be

$$
\begin{equation*}
e_{i}=y-y_{i}=A x_{i}+B-y_{i} \tag{2}
\end{equation*}
$$

To determine the best straight line that fits the $N$, sets of data, $A$ and $B$ have to be chosen so that the sum of the squares of the "errors" is minimized. This means that the simultaneous equations, obtained by equating the partial derivatives of $\left(y-y_{i}\right)^{2}$ with respect to $A$ and $B$ to zero, should be solved. This condition leads then to the following results

$$
\begin{equation*}
A=\frac{N \sum\left(x_{i} y_{i}\right)-\sum x_{i} \sum y_{i}}{\Delta} \tag{3}
\end{equation*}
$$

and

$$
\begin{equation*}
B=\frac{\sum x_{i}^{2} \sum y_{i}-\sum x_{i} \sum\left(x_{i} y_{i}\right)}{\Delta} \tag{4}
\end{equation*}
$$

where

$$
\begin{equation*}
\Delta=N \sum x_{i}^{2}-\left(\sum x_{i}\right)^{2} \tag{5}
\end{equation*}
$$

The correlation coefficient $r$ provides an indicator of how good a fit the best straight line is. This coefficient is defined as

$$
\begin{equation*}
r=\frac{\sum\left(x_{i}-\bar{x}\right)\left(y_{i}-\bar{y}\right)}{\sqrt{\sum\left(x_{i}-\bar{x}\right)^{2} \sum\left(y_{i}-\bar{y}\right)^{2}}} \tag{6}
\end{equation*}
$$

For $r=0$, the values of x and y are independent of one another and there is no linear correlation. The closer $r$ is to +1 or to -1 , the better the linear correlation is.
Finally, the error in A is given by:

$$
\sigma_{A}^{2}=\frac{N}{N-2} \frac{\sum e_{i}^{2}}{\Delta}
$$

## II. Questions

1. (15\%) In the "Michelson Interferometer" experiment,
a- Explain why the interference fringes shown on the screen appear to move when we change the inclination of the glass plate placed between the beam splitter and the mirror. (8\%)
b- In this part of the experiment what variables were measured in order to determine the index of refraction of the glass plate? (7\%)
2. $\mathbf{( 2 0 \% )}$ ) In an attempt to determine the gravitational acceleration experimentally, the motion of an object down an inclined frictionless surface was studied. The time needed to cover a certain distance d was measured. The inclination angle is $\theta=17^{\circ} \pm 0.1 \%$.The following results were obtained:

| t (seconds) | $\mathrm{d}(\mathrm{cm})$ | $\mathrm{a}(\quad)$ |
| :--- | :--- | :--- |
| 0.198 | 6 |  |
| 0.262 | 10 |  |
| 0.286 | 12 |  |
| 0.321 | 14 |  |


a- Determine the average value of the acceleration down the incline along with its error. (10\%)
b- Derive the relationship between the acceleration a, the gravitational acceleration g , and the inclination angle $\theta .(5 \%)$
c- Find g along with its error. (5\%)
3. What are the different parts of a spectrometer and what are the functions of each? (15\%)

