EECE 320 – Digital Systems Design
Homework #1

Exercise 1

The Babylonians developed the *sexagesimal* (base 60) number system about 4000 years ago.

a) How many bits of information is conveyed with one *sexagesimal* digit?

b) How do you write the number 4000_{10} in *sexagesimal*?

Each digit conveys $\log_2 60 = 5.91$ bits of information. $4000_{10} = 1\ 6\ 40_{60}$ (1 in the 3600 column, 6 in the 60's column, and 40 in the 1's column).

Exercise 2

What are the largest and smallest 16-bit binary numbers that can be represented with?

a) unsigned numbers?

b) two's complement numbers?

c) sign/magnitude numbers?

$2^{16}-1 = 65535$; (b) $2^{15}-1 = 32767$; (c) $2^{15}-1 = 32767$

(a) 0; (b) $-2^{15} = -32768$; (c) $-(2^{15}-1) = -32767$

Exercise 3

Convert the following unsigned binary numbers to decimal.

a) 1010

b) 110110

c) 11110000

d) 0001100010100111

(a) 10; (b) 54; (c) 240; (d) 2215

Exercise 4

Convert the following hexadecimal numbers to decimal.

a) $A5_{16}$ $10x16 + 5 = 165$

b) $D0000000_{16}$ $13x16^7 = 3,489,660,928$

Exercise 5

Convert the following two's complement binary numbers to decimal. Repeat for sign/magnitude.

a) 1010

b) 110110

c) 01110000

d) 10011111

(a) 165; (b) 59; (c) 65535; (d) 3489660928

Exercise 6

Convert the following decimal numbers to unsigned binary, then to Octal, and then to Hexadecimal numbers.

a) 42

b) 845

(a) 101010; (b) 1101001101

Exercise 7

Convert the following decimal numbers to 8-bit two's complement numbers or indicate that the decimal number would overflow the range. Repeat for sign/magnitude

a) 42 b) —63 c) 124 d) —128 e) 133

(a) 00101010; (b) 11000001; (c) 01111100; (d) 10000000; (e) overflow

Exercise 8

Convert the following 4-bit two's complement numbers to 8-bit two's complement numbers. Repeat if the numbers were unsigned.

a) 0101

b) 1010

(a) 00000101; (b) 11111010

Exercise 9

a) Convert to octal the binary string: 10101101101001.011011 25551.33

b) Convert to hexadecimal the binary string: 11010110101101010111001.1011010

6B5AB9.B4

Exercise 10

Perform the following additions of unsigned binary numbers. Indicate whether or not the sum overflows an 8-bit result. Repeat if the numbers were in two's complement.

a) 10011001 + 01000100 11011101 same

b) 11010010 + 10110110 overflow; 10001000

Exercise 11

Perform the following additions of unsigned hexadecimal numbers. Indicate whether or not the sum overflows an 8-bit (two hex digit) result.

a) 7 + 9 b) 13 + 28 c) AB + 3E d) 8F + AD

a) 10 b) 3B c) E9 d) 13C, overflow

Exercise 12

Convert the following decimal numbers to 5-bit two's complement binary numbers and subtract them. Indicate whether or not the difference overflows a 5-bit result.

a) 9 - 7 b) 12 - 15 c) - 6 - 11 d) 4 - 8

a) 11010 b) 11100 c) overflow d) 11100

Exercise 13

In a *binary coded decimal* (BCD) system, 4 bits are used to represent a decimal digit from 0 to 9. For example, 37_{10} is written as 00110111_{BCD}.

a) Write 289_{10} in BCD. 0010 1000 1001

b) Convert 100101010001_{BCD} to decimal 951

c) Convert 01101001_{BCD} to binary. 39_{10} 100111_2

d) Represent in **Excess-4** the decimal number: 946 1101 1000 1010

e) Represent in **BCD** the decimal number: 946 1001 0100 0110

Exercise 14

A flying saucer crashes in a Nebraska cornfield. The FBI investigates the wreckage and finds an engineering manual containing an equation in the Martian number system:

325 + 42 = 411

If this equation is correct, how many fingers would you expect Martians have?

$3b^2 + 2b + 5 + 4b + 2 = 4b^2 + b + 1$ ➔ $b^2 - 5b - 6 = 0$ ➔ $b = 6$

Exercise 15

a) The maximum decimal value that can be represented by a 3-digit, base-b system is $124_{(10)}$. What is the value of the base b? (The maximum value in any base with 3 digits is: [(b-1)(b-1)(b-1)])

Maximum value = $b^3 - 1 = 124$ ➔ $b^3 = 125$ ➔ $b = 5$

b) One solution to the quadratic equation $\mathbf{x^2 - 12x + 33 = 0}$ is x = 5. Accordingly, what is the corresponding base used? (Note that if the base is b, then the decimal value of 12 is b+2, of 33 is 3b+3)

$25 - (b+2)5 + 3b+3 = 0$ ➔ $-2b + 18 = 0$ ➔ $b = 9$

Exercise 16

The king receives 64 gold coins in taxes but has reason to believe that one is counterfeit. He summons you to identify the fake coin. You have a balance that can hold coins on each side. How many times do you need to use the balance to find the lighter, fake coin?

$Log_2(64) = 6$.

Exercise 17

Find the Gray code for 4 bits. You can use any algorithm.

0000
0001
0011
0010
0110
0111
0101
0100
1100
1101
1111
1110
1010
1011
1001
1000